

If the loudness of the fundamental ($n = 1$) is $L_1 = 60 \text{ dB}$ (100 dB) for a triangle wave, this corresponds to an intensity associated with the fundamental tone of $I_1 = 10^{-6}$ (10^{-2}) Watts/m^2 , respectively. If the *ratio* of the amplitude for the n^{th} harmonic to the amplitude of the fundamental associated with the triangle wave is $|r_n| / |r_1| = 1/n^2$, for *odd* $n = 3, 5, 7, 9, \dots$ etc. Then the ratio of intensity for the n^{th} harmonic to the intensity for the fundamental associated with the triangle wave is $I_n / I_1 = (1/n)^4$, and the terms, e.g for $n = 3$ are:

$$\log_{10}(I_n / I_1) = \log_{10}(1/n)^4 = 4 \log_{10}(1/n) = 4 \log_{10}(0.3333) = -1.9085$$

and

$$\log_{10}(I_1 / I_o) = 6 \text{ (10)} \quad \text{for} \quad I_1 = 10^{-6} \text{ (} 10^{-2} \text{) Watts/m}^2, \text{ respectively.}$$

Thus, the human ear will perceive the loudness, L_n of the n^{th} harmonic, relative to perceived loudness, L_1 of the fundamental of the triangle wave, as heard e.g. through a loudspeaker as:

$$L_n / L_1 = 1 + \{ \log_{10}(I_n / I_1) / \log_{10}(I_1 / I_o) \}$$

Then for the 3rd harmonic:

$$\begin{aligned} L_3 / L_1 &= 1 - \{ 1.9085 / 6 \} \quad (= 1 - \{ 1.9085 / 10 \}) \\ &= 68.2\% \quad \quad \quad (= 80.9\%) \end{aligned}$$

for $I_1 = 10^{-6}$ (10^{-2}) Watts/m^2 , respectively. This is the (fractional) amount of third harmonic, as heard by the human ear for a triangle wave. This is quite large, but again, not as large as that for the square wave! Again, note that the ratio, L_n / L_1 increases (logarithmically) with increasing amplitude of the square wave! For a loudness of the fundamental tone of $L_1 = 60 \text{ dB}$ (100 dB), the loudness of the third harmonic, for $|r_3| / |r_1| = 1/3 = 33.3\%$ is:

$$\begin{aligned} L_3 &= 10 \log_{10}(I_3 / I_1) + 10 \log_{10}(I_1 / I_o) \\ &= 40 \log_{10}(0.3333) + 60 \text{ dB (} 100 \text{ dB)} \\ &= -19.08 \text{ dB} + 60 \text{ dB (} 100 \text{ dB)} \\ &= 40.92 \text{ dB (} 80.92 \text{ dB)}, \text{ respectively.} \end{aligned}$$

The following figure shows the loudness ratios, L_n / L_1 for the first twenty harmonics (i.e. $n < 20$) associated with the bipolar triangle wave, for loudness values of the fundamental of $L_1 = 60 \text{ dB}$ (\sim quiet) and for $L_1 = 100 \text{ dB}$ (\sim quite loud). This is what the human ear perceives as the loudness of the harmonics relative to that of the fundamental. Note that the decrease in the loudness ratio, L_n / L_1 with increasing harmonic #, n is quite slow.