Thus, $2\alpha_p = \beta_{pick} = 1/8 << 1$, and we can also approximate the factor $\left[{}^{1}/_{(1-2\alpha p)}\right]$ in the above approximate expression for the odd-function Fourier coefficients, b_n by taking the leading terms in the Taylor series expansion for the function $1/(1-\varepsilon)$ for $\varepsilon << 1$:

$$\frac{1}{1-\varepsilon} = 1 + \varepsilon + \varepsilon^2 + \varepsilon^3 + \varepsilon^4 + ... = \sum_{n=1}^{n=\infty} \varepsilon^n \quad for \quad -1 < \varepsilon < 1$$

Thus, for $\varepsilon << 1$, $1/(1-\varepsilon) \sim 1 + \varepsilon$. Thus for $2\alpha_p = 1/8 << 1$, the factor $\left[{}^{1}/_{(1-2\alpha p)}\right] \sim 1 + 2\alpha_p$.

Then, for $n \leq 5$ and $\beta_{pick} = L_{pick} / L_{scale} = ½ / 25 = 0.0200$, we have (approximately) that:

$$b_n \sim 2*\left({}^{1}/_{n\pi}\right) *\left[{}^{1}/_{(1-2\alpha p)}\right] \sim 2*\left({}^{1}/_{n\pi}\right)*\left(1 + 2\alpha_p\right) \sim 2/n\pi$$

This (approximate) result for the low-order harmonic, odd-function Fourier coefficients, b_n, and thus the *magnitudes* of the harmonic amplitudes, $|r_n| = |b_n|$ shows that they decrease as $\sim 1/n$ for the harmonic #, n when picking notes very near to the bridge of the guitar.

However, from the above discussions associated with the bipolar triangle and sawtooth waves, we found, for picking notes e.g. near the mid-point and/or the quarter point on the strings of the guitar, that the harmonic amplitudes, $|r_n| = |b_n|$ decreased as $\sim 1/n^2$ (not as $\sim 1/n$)!!! Therefore, picking notes on the strings very near to the bridge of the guitar, the tone is much brighter there because the low-order harmonics do not fall off in amplitude nearly as fast as they do when playing far away from the bridge!

In the following figure, we show the <u>*exact*</u> (i.e. no approximations-made) results for the magnitudes of the harmonic amplitudes, $|r_n| = |b_n|$ associated with the sawtooth wave for the case when $\beta_{pick} = L_{pick} / L_{scale} = ½ / 25 = 1/50 = 0.0200$.