

harmonics, but differentially, the fundamental is suppressed moreso than the other harmonics, the second harmonic suppressed less, the third harmonic, even less suppressed, and so on, near the bridge, for the first few harmonics, with $\beta_{pick} = L_{pick} / L_{scale} < 1/8$, or equivalently, $\alpha_p = 1/2 \beta_{pick} < 1/16$.

Suppose we decide to pick very close to the bridge, such that the fractional distance, $\beta_{pick} = L_{pick} / L_{scale} \ll 1/8$, corresponding to $\alpha_p = 1/2 \beta_{pick} \ll 1/16$. For definiteness' sake, let us choose $\beta_{pick} = L_{pick} / L_{scale} = 1/2 / 25 = 1/50 = 0.0200 \ll 1/8 = 0.1250$, corresponding to $\alpha_p = 1/2 \beta_{pick} = 1/2 / 50 = 1/100 = 0.0100 \ll 1/16 = 0.0625$.

Now let us look at the generalized expression we obtained above for the *odd*-symmetry Fourier coefficients, b_n associated with the sawtooth wave:

$$b_n = 2 * \left[\frac{2\alpha_p}{(1-2\alpha_p)} \right] * \left(\frac{1}{2n\pi\alpha_p} \right)^2 \sin(2n\pi\alpha_p) \text{ for all } n > 0$$

If we consider only the lower-order harmonics, e.g. $n \leq 5$, then the argument of the *sine* function in the above formula, $(2n\pi\alpha_p) < 2 * 5 * \pi / 100 = \pi / 10 = 0.314159\dots$

Now note that $\sin(\pi/10) = \sin(0.314159\dots) = 0.309017\dots$. The numerical value of $\sin(\pi/10) = 0.309017\dots$ is within $\sim 2\%$ of the argument of the *sine* function, $\pi/10 = 0.314159\dots$. The reason this is so, can be understood from the Taylor series expansion of the *sin* (x) function:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=1}^{n=\infty} \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!}$$

For small values of the argument, x of the *sin* (x) function, e.g. $x \ll 1$, then the higher-order terms in the Taylor series expansion of *sin* (x) are negligible, and thus *sin* (x) $\sim x$ for $x \ll 1$. This also works reasonably well for $x < 1$ (not just $x \ll 1$), as we have seen above, as an approximation.

Thus, for $n \leq 5$ and for $\beta_{pick} = L_{pick} / L_{scale} = 1/2 / 25 = 1/50 = 0.0200$, then

$$\sin(2n\pi\alpha_p) \leq \sin(\pi/10) \sim \pi/10$$

or simply, $\sin(2n\pi\alpha_p) \sim 2n\pi\alpha_p$ for $n \leq 5$ and $\beta_{pick} = L_{pick} / L_{scale} = 1/2 / 25 = 0.0200$.

Then:

$$b_n \sim 2 * \left[\frac{2\alpha_p}{(1-2\alpha_p)} \right] * \left(\frac{1}{2n\pi\alpha_p} \right)^2 2n\pi\alpha_p = 2 * \left[\frac{2\alpha_p}{(1-2\alpha_p)} \right] * \left(\frac{1}{2n\pi\alpha_p} \right)$$

or:

$$b_n \sim 2 * \left(\frac{1}{n\pi} \right) * \left[\frac{1}{(1-2\alpha_p)} \right]$$

Now:

$$\alpha_p = 1/2 \beta_{pick} = 1/2 / 50 = 1/100 = 0.0100 \ll 1/16 = 0.0625.$$