

Using these relations in the above formulae for determining the Fourier coefficients,  $a_n$  and  $b_n$  we obtain, after much algebra and using the fact that  $\sin(3n\pi/2) = -\sin(n\pi/2)$ , that:

$$a_n = 0 \text{ for all } n > 0$$

and:

$$b_n = 2 \cdot (2/n\pi)^2 \sin(n\pi/2)$$

The *even* Fourier coefficients,  $b_n = 0$  for  $n = 2, 4, 6, 8, \dots$  etc.

The *odd* Fourier coefficients,  $b_n = +2 \cdot (2/n\pi)^2$  for  $n = 1, 5, 9, 13, \dots$  etc.

The *odd* Fourier coefficients,  $b_n = -2 \cdot (2/n\pi)^2$  for  $n = 3, 7, 11, 15, \dots$  etc.

Thus, the Fourier series for the symmetrical, bipolar triangle wave of unit amplitude, as shown in the above figure is given by:

$$f(\theta) \Big|_{\substack{\text{triangle} \\ \text{-wave}}} = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos \theta_n + \sum_{n=1}^{n=\infty} b_n \sin \theta_n = 2 \sum_{\substack{n=1 \\ \text{odd}-n}}^{n=\infty} (-1)^{(n-1)/2} \left(\frac{2}{n\pi}\right)^2 \sin(n\theta)$$

Using the replacement:  $n_{\text{odd}} = 2m - 1, m = 1, 2, 3, 4, \dots$  in the above summation, we can alternatively write the Fourier series expansion for this triangle wave as:

$$f(\theta) \Big|_{\substack{\text{triangle} \\ \text{-wave}}} = 2 \sum_{m=1}^{m=\infty} (-1)^{m-1} \left(\frac{2}{(2m-1)\pi}\right)^2 \sin[(2m-1)\theta] = \frac{8}{\pi^2} \left\{ \sin \theta - \frac{1}{9} \sin 3\theta + \frac{1}{25} \sin 5\theta - \frac{1}{49} \sin 7\theta + \dots \right\}$$

Note that the *magnitudes* of the non-zero amplitudes of the harmonics,  $|r_n| = |b_n| = 8/(n\pi)^2$ , as shown in the figure(s) below for the first 20 harmonics.

