

We will again find that due to the intrinsic, overall odd-symmetry of the bipolar sawtooth waveform, that

$$a_n = 0 \text{ for all } n > 0$$

and that:

$$b_n = 2^* \left[\frac{2\alpha_p}{(1-2\alpha_p)} \right]^* \left(\frac{1}{2n\pi\alpha_p} \right)^2 \sin(2n\pi\alpha_p) \text{ for all } n > 0$$

The factor in brackets, $\left[\frac{2\alpha_p}{(1-2\alpha_p)} \right]$ has physical significance - it is the (absolute-value) ratio of the *slope* of the middle portion of the sawtooth waveform to the *slope* of (either) end-portion of the sawtooth waveform, i.e.

$$\left[\frac{2\alpha_p}{(1-2\alpha_p)} \right] = \left| -\left(\frac{1}{\pi(1-2\alpha_p)} \right) / \left(\frac{1}{2\pi\alpha_p} \right) \right|$$

Again, the physically allowed range for α_p is $0 < \alpha_p < 1/2$. Note that the endpoints of this interval are excluded, since both $\alpha_p = 0$ and $\alpha_p = 1/2$ correspond to the *sawtooth* waveform “evolving” into a *ramp* waveform, which physically cannot happen, because of the *boundary-condition* requirement that *each* of the n harmonic waves have *nodes* at the endpoints of the generic interval $0 \leq \theta < 2\pi$ (and at $\theta = \pi$, for the guitar, at the nut). However note mathematically (referring to the above formula for the Fourier coefficient, b_n) that in fact when either $\alpha_p = 0$ and/or $\alpha_p = 1/2$, we discover that $b_n = 0$. Thus, the mathematics tells us, *because* of the boundary conditions, that *no* wave solutions exist for $\alpha_p = 0$ and/or $\alpha_p = 1/2$.

If the value of the α_p -parameter for the peak location(s) of the sawtooth wave is such that $\theta = 2\pi\alpha_p$ and $\theta = 2\pi(1-\alpha_p)$ correspond to peak positions along the sawtooth waveform that coincide with a *node* for a particular harmonic, n , then the Fourier coefficient, b_n will vanish for that harmonic. For physically-allowed values of the α_p -parameter, from the above formula for the Fourier coefficients, b_n we see that a particular Fourier coefficient, b_n will vanish whenever $\sin(2n\pi\alpha_p)$ vanishes, i.e. when $2n\pi\alpha_p = m\pi$ (where the integer $m = 1, 2, 3, \dots$ etc.), i.e. when $n = m/2\alpha_p$, or equivalently, when $\alpha_p = m/2n$ ($< 1/2$).

We have already encountered this phenomenon for the above specific case(s) of the bipolar triangle wave, with $\alpha_p = 1/4$, corresponding to $\theta_p = \pi/2$ and $\theta_p = 3\pi/2$, where *all* of the even- n Fourier harmonics, b_n vanished, because they had nodes at these θ -values; and the case of the bipolar sawtooth wave, with $\alpha_p = 1/8$, corresponding to $\theta_p = \pi/4$ and $\theta_p = 7\pi/4$, where the $n = 4^{\text{th}}, 8^{\text{th}}, 12^{\text{th}}, 16^{\text{th}}, \dots$ etc. Fourier harmonics, b_n vanished, because they too had nodes at these θ -locations.

Thus, for $n \geq 2$ (e.g. $n = 2, 3, 4, 5, 6, \dots$ etc.), whenever the value of the α_p -parameter is such that $\alpha_p = 1/2n$, corresponding to $\theta_p = 2\pi\alpha_p = 2\pi/2n = \pi/n$ and $\theta_p = 2\pi(1-\alpha_p) = 2\pi(1 - 1/2n)$, the Fourier coefficient, b_n will *vanish* for that harmonic associated with the bipolar sawtooth wave.