

Again, we can determine the Fourier coefficients, a_0 , a_n and b_n from the inner products:

$$\begin{aligned} a_0 &= \frac{1}{\pi} \langle f(\theta), 1 \rangle = \frac{1}{\pi} \int_{\theta=\theta_1}^{\theta=\theta_2} f(\theta) d\theta = \frac{1}{\pi} \int_{\theta=0}^{\theta=2\pi} f(\theta) d\theta \\ &= \frac{1}{\pi} \left[\int_{\theta=0}^{\theta=2\pi\alpha} f(\theta) d\theta + \int_{\theta=2\pi\alpha}^{\theta=2\pi(1-\alpha)} f(\theta) d\theta + \int_{\theta=2\pi(1-\alpha)}^{\theta=2\pi} f(\theta) d\theta \right] \\ &= \frac{1}{\pi} \left[\int_{\theta=0}^{\theta=2\pi\alpha} \left(\frac{1}{2\pi\alpha} \right) \theta d\theta + \int_{\theta=2\pi\alpha}^{\theta=2\pi(1-\alpha)} \left(\left(\frac{-1}{\pi(1-2\alpha)} \right) \theta + \frac{1}{1-2\alpha} \right) d\theta + \int_{\theta=2\pi(1-\alpha)}^{\theta=2\pi} \left(\left(\frac{1}{2\pi\alpha} \right) \theta - \frac{1}{\alpha} \right) d\theta \right] \end{aligned}$$

Since this *bipolar* sawtooth waveform has no d.c. offset, we know that $a_0 = 0$.

The Fourier coefficients, a_n and b_n are:

$$\begin{aligned} a_n &= \frac{1}{\pi} \langle f(\theta), \cos(\theta_n) \rangle = \frac{1}{\pi} \int_{\theta=\theta_1}^{\theta=\theta_2} f(\theta) \cos(\theta_n) d\theta = \frac{1}{\pi} \int_{\theta=0}^{\theta=2\pi} f(\theta) \cos(n\theta) d\theta \\ &= \frac{1}{\pi} \left[\int_{\theta=0}^{\theta=2\pi\alpha} f(\theta) \cos(n\theta) d\theta + \int_{\theta=2\pi\alpha}^{\theta=2\pi(1-\alpha)} f(\theta) \cos(n\theta) d\theta + \int_{\theta=2\pi(1-\alpha)}^{\theta=2\pi} f(\theta) \cos(n\theta) d\theta \right] \\ &= \frac{1}{\pi} \left[\int_{\theta=0}^{\theta=2\pi\alpha} \left(\frac{1}{2\pi\alpha} \right) \theta \cos(n\theta) d\theta + \int_{\theta=2\pi\alpha}^{\theta=2\pi(1-\alpha)} \left(\left(\frac{-1}{\pi(1-2\alpha)} \right) \theta + \frac{1}{1-2\alpha} \right) \cos(n\theta) d\theta + \int_{\theta=2\pi(1-\alpha)}^{\theta=2\pi} \left(\left(\frac{1}{2\pi\alpha} \right) \theta - \frac{1}{\alpha} \right) \cos(n\theta) d\theta \right] \end{aligned}$$

and:

$$\begin{aligned} b_n &= \frac{1}{\pi} \langle f(\theta), \sin(\theta_n) \rangle = \frac{1}{\pi} \int_{\theta=\theta_1}^{\theta=\theta_2} f(\theta) \sin(\theta_n) d\theta = \frac{1}{\pi} \int_{\theta=0}^{\theta=2\pi} f(\theta) \sin(n\theta) d\theta \\ &= \frac{1}{\pi} \left[\int_{\theta=0}^{\theta=2\pi\alpha} f(\theta) \sin(n\theta) d\theta + \int_{\theta=2\pi\alpha}^{\theta=2\pi(1-\alpha)} f(\theta) \sin(n\theta) d\theta + \int_{\theta=2\pi(1-\alpha)}^{\theta=2\pi} f(\theta) \sin(n\theta) d\theta \right] \\ &= \frac{1}{\pi} \left[\int_{\theta=0}^{\theta=2\pi\alpha} \left(\frac{1}{2\pi\alpha} \right) \theta \sin(n\theta) d\theta + \int_{\theta=2\pi\alpha}^{\theta=2\pi(1-\alpha)} \left(\left(\frac{-1}{\pi(1-2\alpha)} \right) \theta + \frac{1}{1-2\alpha} \right) \sin(n\theta) d\theta + \int_{\theta=2\pi(1-\alpha)}^{\theta=2\pi} \left(\left(\frac{1}{2\pi\alpha} \right) \theta - \frac{1}{\alpha} \right) \sin(n\theta) d\theta \right] \end{aligned}$$

Again, we will need to use the *indefinite* integrals:

$$\int \cos(n\theta) d\theta = + \frac{\sin(n\theta)}{n}$$

$$\int \sin(n\theta) d\theta = - \frac{\cos(n\theta)}{n}$$

$$\int \theta \cos(n\theta) d\theta = \frac{\cos(n\theta)}{n^2} + \frac{\theta \sin(n\theta)}{n}$$

$$\int \theta \sin(n\theta) d\theta = \frac{\sin(n\theta)}{n^2} - \frac{\theta \cos(n\theta)}{n}$$

And again using the fact(s) that:

$$\sin(2n\pi(1-\alpha_p)) = - \sin(2n\pi\alpha_p)$$

and that:

$$\cos(2n\pi(1-\alpha_p)) = + \cos(2n\pi\alpha_p)$$