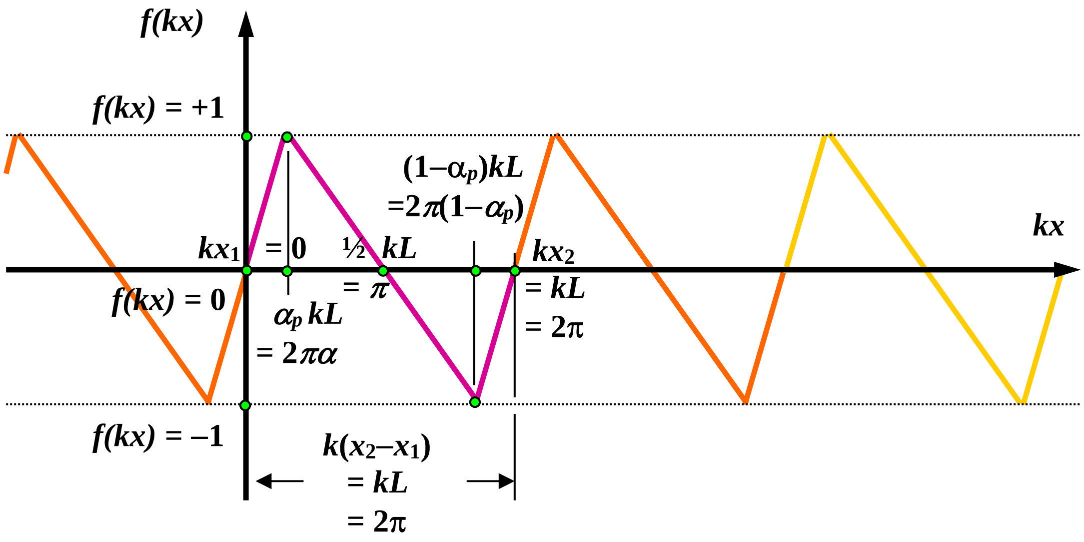
excited. As we mentioned earlier, the fundamental, 2nd and 3rd harmonics are in phase with each other for the sawtooth wave.

F. Fourier Analysis of a Generalized Sawtooth (Asymmetrical Triangle) Wave

We can generalize our above formalism for a bipolar sawtooth (asymmetrical triangle) wave of *unit* amplitude, for a sawtooth wave of any kind, as shown in the figure below:



We introduce the parameter, $\alpha_p \equiv \theta_p/2\pi = kx_p/2\pi = 2\pi x_p/2\pi\lambda = x_p/\lambda$ which physically represents the fractional location of the *first* peak in the sawtooth waveform, located at $\theta_p = kx_p = \alpha_p\, kL = 2\pi\alpha_p$. The parameter α_p <u>*cannot*</u> physically be larger than ½, because the sawtooth wave, $f(\theta)$ <u>*must*</u> be a single-valued function on the "generic" interval $0 \le \theta < 2\pi$, requiring that the first peak in the sawtooth waveform lie within the "generic" interval $0 \le \theta_p < \pi$, which in turn corresponds to a range allowed for the α_p-parameter of $0 < \alpha_p < ½$.

Mathematically, the *odd*-symmetry sawtooth waveform, on the "generic" interval $0 \le \theta < 2\pi$ (i.e. one cycle of this waveform) can then be described in terms of the α_p-parameter as:

$$f(\theta) = f(kx) = +\left(1/2\pi\alpha_p\right)\theta \qquad \text{for} \qquad 0 \le \theta < 2\pi\alpha_p$$

and:

$$f(\theta) = f(kx) = -\left(1/\pi(1-2\alpha_p)\right)\theta + \left(1/(1-2\alpha_p)\right) \quad \text{for} \quad 2\pi\alpha_p \le \theta < 2\pi(1-\alpha_p)$$

and:

$$f(\theta) = f(kx) = +\left(1/2\pi\alpha_p\right)\theta - \left(1/\alpha_p\right) \qquad \text{for} \qquad 2\pi(1-\alpha_p) \le \theta < 2\pi$$

Where we used the straight line equation, $y = mx + b$ to determine the slopes, m and the intercepts, b associated with each of the three line segments in the above waveform on this θ-interval.