

Note that the first four harmonics - the fundamental (aka first harmonic), the second, third and fourth (even though it has zero strength) harmonics all have the same phase angle,  $\delta_n = +180^\circ$ . The next four harmonics have the opposite phase angle,  $\delta_n = -180^\circ$ , the next four after that are in phase again with the first four harmonics, and so on. This behavior of the groups-of-four phase angle arises from the  $\sin(n\pi/4)$  term in the Fourier coefficients,  $b_n$  for the sawtooth waveform.

The *sound* of an audio sawtooth wave to the human ear is *brighter* than the triangle wave, due to the existence of the *second* harmonic in the sawtooth wave, which is *absent* in the triangle wave. If the loudness of the fundamental,  $L_1 = 60 \text{ dB}$  ( $100 \text{ dB}$ ), then the loudness of the second harmonic is  $L_2 = 50.9 \text{ dB}$  ( $91.0 \text{ dB}$ ), corresponding to a loudness ratio of  $L_2/L_1 = 84.9\%$  ( $91.0\%$ ), respectively. For the third harmonic associated with the sawtooth wave,  $L_3 = 40.9 \text{ dB}$  ( $80.9 \text{ dB}$ ), corresponding to a loudness ratio of  $L_3/L_1 = 68.2\%$  ( $80.9\%$ ), respectively. Interestingly enough, these loudness results for the third harmonic of the sawtooth wave are also *precisely* those for the triangle wave, as are all the odd- $n$  loudness results! The sawtooth wave differs from the triangle wave primarily because of the additional presence of the even- $n$  harmonics, however note also that the *phase relations* for the odd- $n$  harmonics are *not* the same for these two waves. As we have mentioned before, the human ear is *not* sensitive to such phase relations.

The following figure shows the loudness ratios,  $L_n/L_1$  for the first twenty harmonics (i.e.  $n < 20$ ) associated with the bipolar sawtooth wave, for loudness values of the fundamental of  $L_1 = 60 \text{ dB}$  ( $\sim$  quiet) and for  $L_1 = 100 \text{ dB}$  ( $\sim$  quite loud). This is what the human ear perceives as the loudness of the harmonics relative to that of the fundamental. Note that the decrease in the loudness ratio,  $L_n/L_1$  with increasing harmonic #,  $n$  is again rather slow.

