

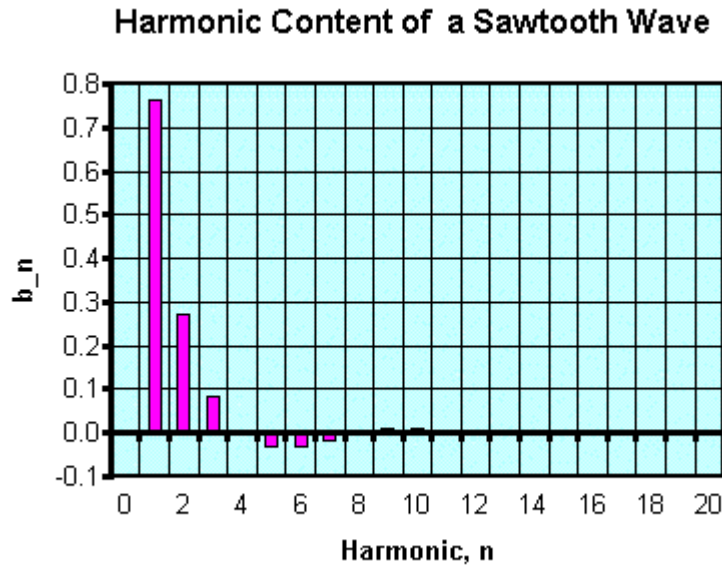
All of the *even*-reflection symmetry Fourier coefficients, $a_n = 0$ because the sawtooth waveform has overall *odd*-reflection symmetry.

Thus, for the sawtooth form of a triangle wave, *both* even- n and odd- n b_n -harmonics are present! The reason for this is that while the *overall* sawtooth waveform still has *odd* reflection symmetry about its midpoint ($\theta = \pi$), i.e. that for $0 \leq \theta \leq 2\pi$, $f(\theta > \pi) = -f((2\pi - \theta) < \pi)$, the sawtooth waveform no longer has any *local* reflection symmetry properties about its peaks - e.g. about $\theta = \pi/4$ and/or about $\theta = 7\pi/4$, i.e. locally, for $0 \leq \theta \leq \pi/2$, $f(\theta > \pi/4) \neq \pm f((\pi/2 - \theta) < \pi/4)$, and for $3\pi/2 \leq \theta \leq 2\pi$, $f(\theta > 7\pi/4) \neq \pm f((2\pi - \theta) < 7\pi/4)$. Because of this, both odd- n and even- n terms in the Fourier coefficients, b_n are needed for the overall odd-reflection symmetry $\sin(n\theta)$ functions associated with the Fourier series expansion for the bipolar sawtooth waveform.

The Fourier series for the bipolar sawtooth wave of unit amplitude, is thus given by:

$$f(\theta) \Big|_{\substack{\text{sawtooth} \\ \text{-wave}}} = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos \theta_n + \sum_{n=1}^{n=\infty} b_n \sin \theta_n = \frac{2}{3} \sum_{n=1}^{n=\infty} \left(\frac{4}{n\pi}\right)^2 \sin\left(\frac{n\pi}{4}\right) \sin(n\theta)$$

The numerical values of the Fourier coefficients, b_n for the bipolar sawtooth wave are shown in the figure below for the first 20 harmonics.



The *magnitudes* of the amplitudes of the harmonics, $|r_n| = |b_n|$ for the bipolar sawtooth wave, again decrease with increasing harmonic #, n , as $\sim 1/n^2$, as for the bipolar triangle wave. We show the numerical values of the $|r_n|$ for the first 20 harmonics of the bipolar sawtooth wave in the figure below. Note that this is a *semi-log* plot of $|r_n|$ vs. n .