

We determine the Fourier coefficients,  $a_0$ ,  $a_n$  and  $b_n$  from the inner products:

$$\begin{aligned}
 a_0 &= \frac{1}{\pi} \langle f(\theta), 1 \rangle = \frac{1}{\pi} \int_{\theta=\theta_1}^{\theta=\theta_2} f(\theta) d\theta = \frac{1}{\pi} \int_{\theta=0}^{\theta=2\pi} f(\theta) d\theta \\
 &= \frac{1}{\pi} \left[ \int_{\theta=0}^{\theta=\pi/2} f(\theta) d\theta + \int_{\theta=\pi/2}^{\theta=3\pi/2} f(\theta) d\theta + \int_{\theta=3\pi/2}^{\theta=2\pi} f(\theta) d\theta \right] \\
 &= \frac{1}{\pi} \left[ \int_{\theta=0}^{\theta=\pi/2} \left(\frac{2}{\pi}\right)\theta d\theta + \int_{\theta=\pi/2}^{\theta=3\pi/2} \left(\left(\frac{-2}{\pi}\right)\theta + 2\right) d\theta + \int_{\theta=3\pi/2}^{\theta=2\pi} \left(\left(\frac{2}{\pi}\right)\theta - 4\right) d\theta \right] \\
 &= \frac{1}{\pi} \left[ \left(\frac{2}{\pi}\right) \frac{1}{2} \theta^2 \Big|_0^{\pi/2} + \left[\left(\frac{-2}{\pi}\right) \frac{1}{2} \theta^2 + 2\theta\right] \Big|_{\pi/2}^{3\pi/2} + \left[\left(\frac{2}{\pi}\right) \frac{1}{2} \theta^2 - 4\theta\right] \Big|_{3\pi/2}^{2\pi} \right] \\
 &= \frac{1}{\pi} \left[ \frac{1}{\pi} \left[ \frac{1}{4} \pi^2 - \frac{9}{4} \pi^2 + \frac{1}{4} \pi^2 + 4\pi^2 - \frac{9}{4} \pi^2 \right] + [3\pi - \pi - 8\pi + 6\pi] \right] = 0
 \end{aligned}$$

Since this waveform is *bipolar*, it has no d.c. offset, thus  $a_0 = 0$ .

The Fourier coefficients,  $a_n$  and  $b_n$  for  $n > 0$  are:

$$\begin{aligned}
 a_n &= \frac{1}{\pi} \langle f(\theta), \cos(\theta_n) \rangle = \frac{1}{\pi} \int_{\theta=\theta_1}^{\theta=\theta_2} f(\theta) \cos(\theta_n) d\theta = \frac{1}{\pi} \int_{\theta=0}^{\theta=2\pi} f(\theta) \cos(n\theta) d\theta \\
 &= \frac{1}{\pi} \left[ \int_{\theta=0}^{\theta=\pi/2} f(\theta) \cos(n\theta) d\theta + \int_{\theta=\pi/2}^{\theta=3\pi/2} f(\theta) \cos(n\theta) d\theta + \int_{\theta=3\pi/2}^{\theta=2\pi} f(\theta) \cos(n\theta) d\theta \right] \\
 &= \frac{1}{\pi} \left[ \int_{\theta=0}^{\theta=\pi/2} \left(\frac{2}{\pi}\right)\theta \cos(n\theta) d\theta + \int_{\theta=\pi/2}^{\theta=3\pi/2} \left(\left(\frac{-2}{\pi}\right)\theta + 2\right) \cos(n\theta) d\theta + \int_{\theta=3\pi/2}^{\theta=2\pi} \left(\left(\frac{2}{\pi}\right)\theta - 4\right) \cos(n\theta) d\theta \right]
 \end{aligned}$$

$$\begin{aligned}
 b_n &= \frac{1}{\pi} \langle f(\theta), \sin(\theta_n) \rangle = \frac{1}{\pi} \int_{\theta=\theta_1}^{\theta=\theta_2} f(\theta) \sin(\theta_n) d\theta = \frac{1}{\pi} \int_{\theta=0}^{\theta=2\pi} f(\theta) \sin(n\theta) d\theta \\
 &= \frac{1}{\pi} \left[ \int_{\theta=0}^{\theta=\pi/2} f(\theta) \sin(n\theta) d\theta + \int_{\theta=\pi/2}^{\theta=3\pi/2} f(\theta) \sin(n\theta) d\theta + \int_{\theta=3\pi/2}^{\theta=2\pi} f(\theta) \sin(n\theta) d\theta \right] \\
 &= \frac{1}{\pi} \left[ \int_{\theta=0}^{\theta=\pi/2} \left(\frac{2}{\pi}\right)\theta \sin(n\theta) d\theta + \int_{\theta=\pi/2}^{\theta=3\pi/2} \left(\left(\frac{-2}{\pi}\right)\theta + 2\right) \sin(n\theta) d\theta + \int_{\theta=3\pi/2}^{\theta=2\pi} \left(\left(\frac{2}{\pi}\right)\theta - 4\right) \sin(n\theta) d\theta \right]
 \end{aligned}$$

Now the *indefinite* integrals:

$$\int \cos(n\theta) d\theta = + \frac{\sin(n\theta)}{n}$$

$$\int \sin(n\theta) d\theta = - \frac{\cos(n\theta)}{n}$$

$$\int \theta \cos(n\theta) d\theta = \frac{\cos(n\theta)}{n^2} + \frac{\theta \sin(n\theta)}{n}$$

$$\int \theta \sin(n\theta) d\theta = \frac{\sin(n\theta)}{n^2} - \frac{\theta \cos(n\theta)}{n}$$