Note that the *ratio* of harmonic amplitudes for this square wave, relative to the fundamental is $|r_n| / |r_1| = 1/n$, which does not decrease very fast, as the harmonic #, $n$ increases ($n$ = 3, 5, 7, 9, ... etc.).

As far as harmonic content goes, *any* kind of square wave, compared to just about any other kind of waveform is *extremely* rich in harmonics. The reason for this is due to the very sharp "breaks" or "jumps" (i.e. the discontinuities) in the waveform. To *make* such sharp edges in the waveform, extremely high harmonics, with correspondingly very short wavelengths are needed, even though their relative amplitudes may be small.

The human ear hears a square-wave audio signal as being very "bright", relative to e.g. a pure-tone (sine-wave) audio signal at the same frequency, which sounds "mellow" or "round", since it has only a single harmonic component - the fundamental. In fact, the square wave audio signal also sounds "harsh" to the human ear, because of the presence of all of the *odd* harmonics, at 3*f*, 5*f*, 7*f*, 9*f*, .... etc.

Note that harmonics at the frequencies 3*f*, 5*f*, 7*f*, 9*f*, .... etc. are not integer-multiples of an octave above the fundamental, at frequency, *f*. A frequency that is one octave above the fundamental is at 2*f*; two octaves above, at 4*f*; three octaves above, at 6*f*, ... etc.

If the fundamental is at a frequency, $f = 440$ *Hz* (i.e. $A_4$ on a piano), then $3f = 1320$ *Hz* (very close to $E_6$ on a piano), which is one octave and a *fifth* above the fundamental. The fifth harmonic is $5f = 2200$ *Hz* (very close to $C_7^{\#}$ on a piano), which is two octaves and a *third* above the fundamental. Together, ignoring the octaves, these two harmonics, in combination with the fundamental, form a major triad-type chord (in the key of *A*, here), so it isn't *that* displeasing to the human ear to listen to a square wave-type of sound.

If a square wave signal, e.g. created by a function generator is output through a loudspeaker, converting it to sound, the human ear perceives the *loudness*, *L* of this sound (units of *deci-Bels*, abbreviated as *dB*) which is *logarithmically* proportional to the *intensity*, *I* ) of the sound wave (units of $Watts/m^2$), which in turn is linearly proportional power, *P* of the sound wave (units of *Watts*), which in turn is proportional to the *square* of the amplitude, $A_i$ of the square wave. Mathematically:

$$Loudness,\ L \equiv 10\ log_{10}(I / I_o) \quad \text{(units = } deci\text{-}Bels,\ dB\text{)}$$

$$Intensity,\ I\ (Watts/m^2) \propto Power,\ P\ (Watts) \propto \{Output\ Response,\ R_o(S_i(t))\}^2$$

The *threshold* of human hearing - i.e. the faintest possible sound that is detectable as such by the (average) human ear is defined as *Loudness*, $L_o \equiv 0\ dB$, which corresponds to a sound intensity, $I_o$ associated with the threshold of human hearing of $I_o = 10^{-12}\ Watts/m^2$.

If the loudness of the fundamental ($n = 1$) is $L_1 = 60\ dB\ (100\ dB)$, this corresponds to an intensity associated with the fundamental tone of $I_1 = 10^{-6}\ (10^{-2})\ Watts/m^2$, respectively. If the *ratio* of the amplitude for the $n^{th}$ harmonic to the amplitude of the fundamental associated with the square wave is $|r_n| / |r_1| = 1/n$, for *odd* $n$ = 3, 5, 7, 9, ... etc. Then the ratio of intensity for the $n^{th}$ harmonic to the intensity for the fundamental associated with the square wave is $I_n / I_1 = (1/n)^2$, and the terms, e.g for $n = 3$ are: