If we had shifted the *phase* of the original periodic, bipolar, 50% duty-cycle square wave by e.g. $\Delta\theta = \pm\,\pi/2 = \pm\,90^o$, then this would change the odd-symmetry nature of the waveform to even symmetry. Mathematically, this even-symmetry square wave would be described as:

$$f(\theta) = f(\omega t) = -1 \quad \text{for} \quad 0 \le \theta < \pi/2$$

and:

$$f(\theta) = f(\omega t) = +1 \quad \text{for} \quad \pi/2 \le \theta < 3\pi/2$$

and:

$$f(\theta) = f(\omega t) = -1 \quad \text{for} \quad 3\pi/2 \le \theta < 2\pi$$

The Fourier coefficients for this even-symmetry waveform would be $a_0 = 2$, *all* even-n Fourier coefficients, $a_n = 0$, for $n = 2, 4, 6$, ... etc., but *all* odd-n Fourier coefficients, $a_n = 4/n\pi$ for $n = 1, 3, 5, 7$, ... etc. and *all* Fourier coefficients, $b_n = 0$ for *all* $n = 1, 2, 3, 4, 5, 6$, ... etc.

If we had shifted the phase of the original periodic, bipolar, 50% duty-cycle square wave by e.g. a random shift, $\Delta\theta$, then very likely the resulting waveform would have *neither* odd *nor* even reflection symmetry properties (unless $\Delta\theta$ happened to be e.g. $\pm\,\pi/2$ or $\pm\,\pi$). Mathematically, this waveform would be described as:

$$f(\theta) = f(\omega t) = -1 \quad \text{for} \quad 0 \le \theta < \Delta\theta$$

and:

$$f(\theta) = f(\omega t) = +1 \quad \text{for} \quad \Delta\theta \le \theta < \Delta\theta + \pi$$

and:

$$f(\theta) = f(\omega t) = -1 \quad \text{for} \quad \Delta\theta + \pi \le \theta < 2\pi$$

For a waveform that has *no* reflection symmetry properties whatsoever, in general *all* of the Fourier coefficients, a_0, the a_n and b_n coefficients will be non-zero.

Because of the existence of reflection symmetries in a waveform, certain of the Fourier coefficients, a_0, the a_n and/or b_n will vanish.

However, even for “no-symmetry” waveforms, as we have discussed above, for each harmonic, n of the fundamental, there is physically only *one* amplitude, $|r_n| = (a_n^2 + b_n^2)^{1/2}$ and *one* phase angle, $\delta_n = \tan^{-1}(b_n / a_n)$ (or equivalently $\delta_n' = \tan^{-1}(a_n / b_n)$) associated with that harmonic.

If the *duty-cycle* of the waveform is varied from its “symmetrical” value of 50%, this will have a corresponding impact on *all* of the Fourier coefficients, a_0, the a_n and/or b_n, e.g. since the d.c. value of e.g. a 10% duty-cycle bipolar square wave certainly is not 0! We will discuss this case further, below.

For values of the duty cycle other than “easy” choices of integer fractions of the full cycle in the “generic” theta variable (i.e. $\Delta\theta = \theta_2 - \theta_1 = 2\pi$), the evaluation of the inner products used to determine the Fourier coefficients, a_0, the a_n and b_n can be *very* tedious to carry out by hand. However, these *are* straightforward to carry out on a computer, using e.g. numerical integration techniques.