Since the *even* Fourier coefficients, $a_n = 0$ for *all* harmonics ($n \geq 0$) of this waveform, then the *magnitudes* of the Fourier amplitudes, $|r_n|$ associated with the *odd* harmonics are $|r_n| = (a_n^2 + b_n^2)^{½} = b_n = 4/n\pi$ for *odd* $n = 1, 3, 5, \ldots$ etc., i.e. $|r_{2m-1}| = b_{2m-1} = -4/(2m-1)\pi$ for $m = 1, 2, 3, 4, 5\ldots$ etc. Note that *all* phase angles for these *odd* harmonics are $\delta_n = tan^{-1}(b_n / a_n) = tan^{-1}(\infty) = ½\,\pi = 90^{\text{o}}$ (or equivalently, $\delta_n' = tan^{-1}(a_n / b_n) = tan^{-1}(0) = 0 = 0^{\text{o}}$, since $\delta_n' = \pi/2 - \delta_n = 90^{\text{o}} - \delta_n$).

Only the *odd* harmonics are present in the periodic, bipolar, 50% duty-cycle square wave (as drawn in the figure above) because this waveform, as we have discussed above, has intrinsically *odd* reflection symmetry properties! Thus, simply recognizing the symmetry properties of a waveform instantly tells one which harmonics of the fundamental will or will not be present! Note that the use of symmetry arguments very often is extremely powerful and helpful in terms of gaining insight into the behavior of a physical system!

If we had <u>*flipped*</u> the *polarity* of the waveform, such that initially the waveform was *negative* during the first half of its cycle, then *positive* during the second half of its cycle, this waveform would *still* have odd symmetry, and thus *still* contain the same odd harmonics. However, for this waveform, the *sign* of the non-zero, odd Fourier coefficients, b_n would *reverse* - i.e. $b_n = -4/n\pi$, for odd $n = 1, 3, 5, \ldots$ etc. One can see this by inspection of the details of working out the above inner production computation for the determination of the b_n Fourier coefficients, as well as from the use of reflection symmetry arguments.

If we had *shifted* the *offset* (e.g. by one unit) of the original periodic, bipolar, 50% duty-cycle square wave, such as to make this waveform a *unipolar* square wave, by adding a d.c. offset (i.e. constant term) to the waveform, then this would only affect the a_0 term in the Fourier series expansion of the waveform. For an upward-shifted unipolar square wave of unit amplitude, for one cycle, the mathematical description of such a wave is given by:

$$f(\theta) = f(\omega t) = +2 \quad \text{for} \quad 0 \leq \theta < \pi$$

and:

$$f(\theta) = f(\omega t) = 0 \quad \text{for} \quad \pi \leq \theta < 2\pi$$

The corresponding $n = 0$ Fourier coefficient for this waveform is $a_0 = 2$. The mean, or average value of $f(\theta)$, averaging over one cycle of this *unipolar*, 50% duty-cycle square wave is $\langle f(\theta) \rangle = 1$ ($= a_0/2 = 2/2 = 1$).

Note that if we had used a periodic, bipolar, 50% duty-cycle square wave which had an amplitude of A_i (instead of a *unit* amplitude), then from the inner product computation of the odd Fourier coefficients, b_n we would instead have obtained $b_n = 4A_i/n\pi$ for the odd harmonics, $n = 1, 3, 5, 7, \ldots$ etc. Since mathematically, such a waveform would be defined as:

$$f(\theta) = f(\omega t) = +A_i \quad \text{for} \quad 0 \leq \theta < \pi$$

and:

$$f(\theta) = f(\omega t) = -A_i \quad \text{for} \quad \pi \leq \theta < 2\pi$$