Now the *indefinite* integrals:

$$\int \cos(n\theta)d\theta = +\frac{\sin(n\theta)}{n} \qquad \int \sin(n\theta)d\theta = -\frac{\cos(n\theta)}{n}$$

Thus, the Fourier coefficients, a_n and b_n, for $n > 0$ are:

$$\begin{aligned} a_n &= \frac{+1}{n\pi}\left\{\left[\sin(n\theta)|_{\theta=\pi} - \sin(n\theta)|_{\theta=0}\right] - \left[\sin(n\theta)|_{\theta=2\pi} - \sin(n\theta)|_{\theta=\pi}\right]\right\} \\ &= \frac{+1}{n\pi}\left\{\left[\sin(n\pi) - \sin(0)\right] - \left[\sin(2n\pi) - \sin(n\pi)\right]\right\} = \frac{1}{n\pi}\left\{\left[0-0\right] - \left[0-0\right]\right\} = 0 \end{aligned}$$

since $sin(0) = sin(n\pi) = sin(2n\pi) = 0$ for *all* integers, $n = 1, 2, 3, 4,.....$, and:

$$\begin{aligned} b_n &= \frac{-1}{n\pi}\left\{\left[\cos(n\theta)|_{\theta=\pi} - \cos(n\theta)|_{\theta=0}\right] - \left[\cos(n\theta)|_{\theta=2\pi} - \cos(n\theta)|_{\theta=\pi}\right]\right\} \\ &= \frac{-1}{n\pi}\left\{\left[\cos(n\pi) - \cos(0)\right] - \left[\cos(2n\pi) - \cos(n\pi)\right]\right\} = \frac{-1}{n\pi}\left\{\left[\cos(n\pi) - 1\right] - \left[1 - \cos(n\pi)\right]\right\} \\ &= \frac{-2}{n\pi}\left[\cos(n\pi) - 1\right] = \frac{2}{n\pi}\left[1 - \cos(n\pi)\right] \end{aligned}$$

Now $cos(0) = cos(2n\pi) = +1$ for *<u>all</u>* integers, $n = 1, 2, 3, 4,.....$, and $cos(n\pi) = +1$ for the *<u>even</u>* integers, $n = 2, 4, 6, 8, ...$, and $cos(n\pi) = -1$ for the *<u>odd</u>* integers, $n = 1, 3, 5, 7, ...$

Thus, we see that *<u>all</u>* of the Fourier coefficients, a_n for the *<u>even</u>* functions, $cos(\theta_n)$ vanish i.e. $a_n = 0$ for *<u>all</u>* integers, $n = 1, 2, 3, 4, ...$

The Fourier coefficients, b_n for the *<u>odd</u>* functions, $sin(\theta_n)$ vanish for the *<u>even</u>* harmonics, i.e. $b_n = 0$ when $n = 2, 4, 6, 8, ...$, but the Fourier coefficients, b_n are non-zero for the *<u>odd</u>* harmonics, when $n = 1, 3, 5, 7, ...$, where $b_n = +4/n\pi$.

Thus, the Fourier series expansion of a periodic, bipolar, 50% duty-cycle square wave as shown in the above figure is given by:

$$f(\theta)|_{\substack{square\\-wave}} = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos\theta_n + \sum_{n=1}^{n=\infty} b_n \sin\theta_n = \frac{4}{\pi}\sum_{\substack{n=1\\odd-n}}^{n=\infty} \frac{\sin(n\theta)}{n}$$

Using the replacement: $n_{odd} = 2\,m - 1$, $m = 1, 2, 3, 4, ...$ in the above summation, we can alternatively write the Fourier series expansion for this square wave as:

$$f(\theta)|_{\substack{square\\-wave}} = \frac{4}{\pi}\sum_{m=1}^{m=\infty} \frac{\sin[(2m-1)\theta]}{(2m-1)} = \frac{4}{\pi}\left\{\sin\theta + \tfrac{1}{3}\sin 3\theta + \tfrac{1}{5}\sin 5\theta + \tfrac{1}{7}\sin 7\theta + ...\right\}$$

Thus, we see that for the periodic, bipolar, 50% duty-cycle square wave, only *odd* harmonics (i.e. *odd* integer multiples of the fundamental) are present in this waveform.