

Exercises:

1. Compute the Fourier coefficients, a_0 , a_n and b_n for the “flipped” bipolar, 50% duty-cycle square wave, in the time domain:

$$f(\theta) = f(\omega t) = -1 \quad \text{for } 0 \leq \theta < \pi$$

$$f(\theta) = f(\omega t) = +1 \quad \text{for } \pi \leq \theta < 2\pi$$

Compare these Fourier coefficients with those obtained above for the “unflipped” bipolar, 50% duty-cycle square wave.

2. Compute the Fourier coefficients, a_0 , a_n and b_n for the “shifted” bipolar, 50% duty-cycle square wave, in the time domain:

$$f(\theta) = f(\omega t) = -1 \quad \text{for } 0 \leq \theta < \pi/2$$

$$f(\theta) = f(\omega t) = +1 \quad \text{for } \pi/2 \leq \theta < 3\pi/2$$

$$f(\theta) = f(\omega t) = -1 \quad \text{for } 3\pi/2 \leq \theta < 2\pi$$

Compare these Fourier coefficients with those obtained above for the “unflipped” and “flipped” bipolar, 50% duty-cycle square waves.

3. Compute the Fourier coefficients, a_0 , a_n and b_n for the unipolar, 25% duty-cycle square wave, in the time domain:

$$f(\theta) = f(\omega t) = 0 \quad \text{for } 0 \leq \theta < \pi/2$$

$$f(\theta) = f(\omega t) = +1 \quad \text{for } \pi/2 \leq \theta < 2\pi$$

Compare these Fourier coefficients with those obtained above for the “unflipped” bipolar, 50% duty-cycle square wave.

4. Compute the Fourier coefficients, a_0 , a_n and b_n for the unipolar delta-function waveform, in the time domain:

$$f(\theta) = f(\omega t) = \delta(\theta - \pi)$$

Compare these Fourier coefficients with those obtained above for the bipolar delta-function waveform.

5. For each of the above exercises, use e.g. *MathLab*, or a spreadsheet program, such as *Excel* to make plots of the harmonic amplitudes, $|r_n|$, the loudness ratios, L_n/L_1 and Fourier construction of the original waveform, for e.g. the first few harmonics.

References for Fourier Analysis and Further Reading:

1. Fourier Series and Boundary Value Problems, 2nd Edition, Ruel V. Churchill, McGraw-Hill Book Company, 1969.
2. Mathematics of Classical and Quantum Physics, Volumes 1 & 2, Frederick W. Byron, Jr. and Robert W. Fuller, Addison-Wesley Publishing Company, 1969.
3. Mathematical Methods of Physics, 2nd Edition, Jon Matthews and R.L. Walker, W.A. Benjamin, Inc., 1964.