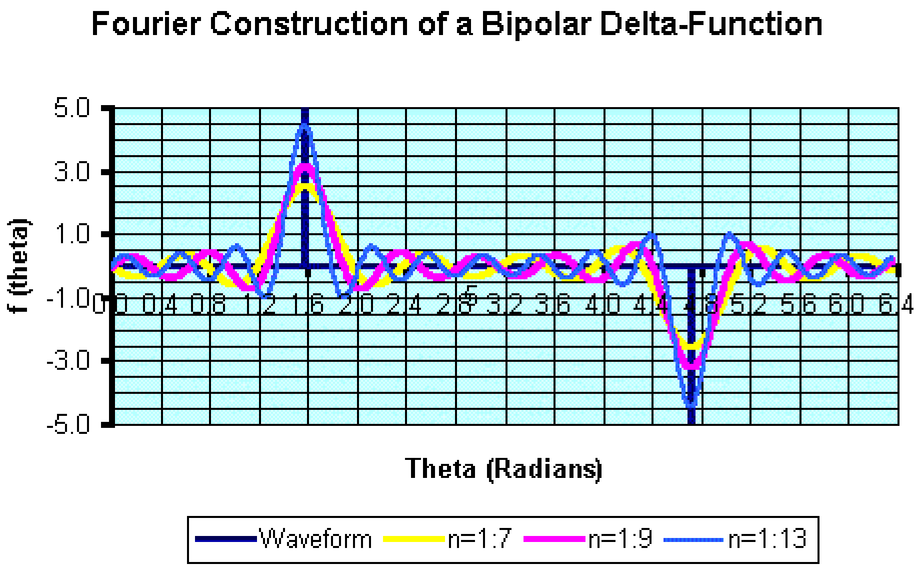

It can be seen that the higher-order harmonics are much needed for decreasing the *width* of the "pulse" at each delta-function location. The width of each pulse slowly decreases as the number of harmonics included in the "Fourier construction" of the bipolar delta function increases. Only in the limit of using an *infinite* number of harmonics does the width of each delta-function "pulse" formally become zero.

The periodic, bipolar delta-function waveform is the "0% duty-cycle" limiting case of the periodic, bipolar 50% duty-cycle square wave. While the harmonic content of the 50% duty-cycle bipolar square wave is already extremely rich in *odd* harmonics, with harmonic amplitudes that decrease slowly, as $1/n$ of the harmonic #, n, the "0% duty-cycle" delta-function waveform is *the extreme* in harmonic content, since all harmonics have the *same* amplitude. For bipolar square waves with duty-cycle (DC) between 0% < DC < 50%, the decrease in harmonic content with increasing harmonic # is less steep than $1/n$, becoming *flatter* with increasing harmonic # as the duty cycle decreases from 50%, to the limiting case for DC = 0%, when the harmonic content with increasing harmonic # is perfectly flat.

If the duty-cycle of the periodic, bipolar square wave increases *beyond* 50%, then the only way this can occur is if the waveform develops a d.c. offset. Thus, the Fourier series for such waveforms develops a non-zero value of a_0 (i.e. $|r_0|$) for DC > 50%. For the limiting case of a bipolar, *unit-amplitude* square wave with duty factor, DC = 100%, then the time average of this waveform, $< f(\theta) > = 1 = a_0 / 2$, thus $a_0 = 2$ here. Note that this 100% duty-cycle waveform is also a periodic, but unipolar (i.e. *single*) delta-function waveform, for *each* cycle of the waveform. The 100% duty cycle, unit-amplitude periodic waveform can thus be thought of as a superposition (i.e. linear combination) of a d.c. offset (of strength $a_0 = 2$) with a periodic, unipolar delta-function waveform. Thus, this waveform will also have a perfectly flat harmonic spectrum, neglecting the zero-frequency d.c. offset term.

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