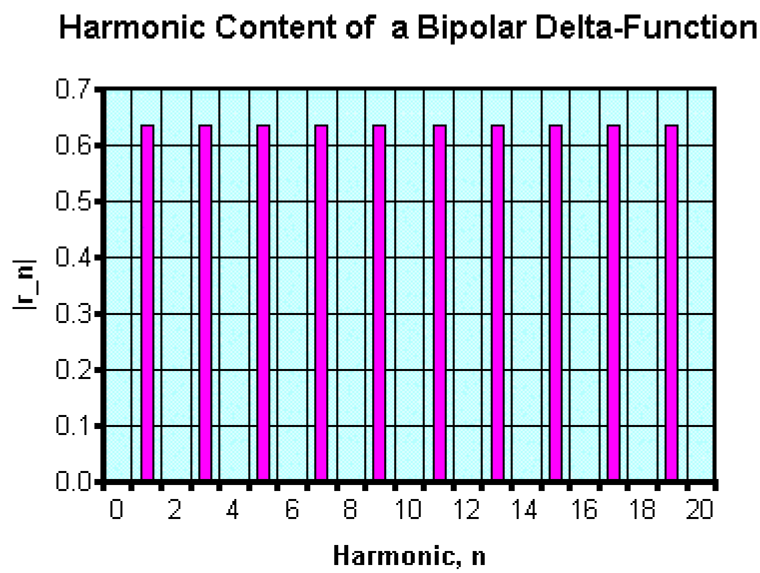
Thus, the Fourier series expansion for a periodic, bipolar, delta-function wave as shown in the above figure is given by:

$$f(\theta)\,|_{\substack{\delta-fcn\\-wave}} = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos\theta_n + \sum_{n=1}^{n=\infty} b_n \sin\theta_n = \frac{2}{\pi}\sum_{\substack{n=1\\odd-n}}^{n=\infty} (-1)^{(n-1)/2}\sin(n\theta)$$

Using the replacement: $n_{odd} = 2m - 1$, m = 1, 2, 3, 4, ... in the above summation, we can alternatively write the Fourier series expansion for this delta-function wave as:

$$f(\theta)\,|_{\substack{\delta-fcn\\-wave}} = \frac{2}{\pi}\sum_{m=1}^{m=\infty} (-1)^{m-1}\sin[(2m-1)\theta] = \frac{2}{\pi}\left\{\sin\theta - \sin 3\theta + \sin 5\theta - \sin 7\theta + \sin 9\theta - .....\right\}$$

Note that the *magnitudes* of the non-zero amplitudes of the harmonics, $|r_n| = |b_n| = 2/\pi$, as shown in the figure below for the first 20 harmonics.



Note that the $|r_n|$ have no n-dependence - i.e. they are independent of frequency! Thus for a bipolar delta-function waveform, <u>*all*</u> *odd* harmonics contribute <u>equally</u> in magnitude to creating this waveform!

However, because the *sign* of the b_n changes with successive *odd* integer, n, this also means that the phase angle, δ_n changes sign with successive *odd* integer, n. For *odd* n = 1, 5, 9, 13, ... etc., where $b_n = +2/\pi$ and $a_n = 0$, then $\delta_n = \tan^{-1}(b_n / a_n) = \tan^{-1}(\infty) = ½\,\pi = 90^o$. For *odd* n = 3, 7, 11, 15, ... etc., where $b_n = -2/\pi$ and $a_n = 0$, then $\delta_n = \tan^{-1}(b_n / a_n) = \tan^{-1}(-\infty) = -½\,\pi = -90^o$. Thus, the non-zero, successive odd-n phase angles, δ_n of the harmonics are 180^o degrees apart - i.e. successive harmonics tend to *cancel* against each other, *except* in the regions $\theta \sim \pi/2$ and $\theta \sim 3\pi/2$!