We can then determine the Fourier coefficients, $a_0$, the $a_n$ and $b_n$ from their associated inner products:

$$a_0 = \frac{1}{\pi}\langle f(\theta),1\rangle = \frac{1}{\pi}\int_{\theta=\theta_1}^{\theta=\theta_2} f(\theta)d\theta = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} f(\theta)d\theta = \frac{1}{\pi}\left[\int_{\theta=0}^{\theta=\pi} f(\theta)d\theta + \int_{\theta=\pi}^{\theta=2\pi} f(\theta)d\theta\right]$$
$$= \frac{1}{\pi}\left[\int_{\theta=0}^{\theta=\pi}\delta(\theta-\tfrac{\pi}{2})d\theta - \int_{\theta=\pi}^{\theta=2\pi}\delta(\theta-\tfrac{3\pi}{2})d\theta\right] = \frac{1}{\pi}[1-1] = 0$$

The Fourier coefficients, $a_n$ and $b_n$ for $n > 0$ are:

$$a_n = \frac{1}{\pi}\langle f(\theta),\cos(\theta_n)\rangle = \frac{1}{\pi}\int_{\theta=\theta_1}^{\theta=\theta_2} f(\theta)\cos(\theta_n)d\theta = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} f(\theta)\cos(n\theta)d\theta$$
$$= \frac{1}{\pi}\left[\int_{\theta=0}^{\theta=\pi} f(\theta)\cos(n\theta)d\theta + \int_{\theta=\pi}^{\theta=2\pi} f(\theta)\cos(n\theta)d\theta\right]$$
$$= \frac{1}{\pi}\left[\int_{\theta=0}^{\theta=\pi}\delta(\theta-\tfrac{\pi}{2})\cos(n\theta)d\theta - \int_{\theta=\pi}^{\theta=2\pi}\delta(\theta-\tfrac{3\pi}{2})\cos(n\theta)d\theta\right] = \frac{1}{\pi}\left[\cos(\tfrac{n\pi}{2}) - \cos(\tfrac{3n\pi}{2})\right]$$

$$b_n = \frac{1}{\pi}\langle f(\theta),\sin(\theta_n)\rangle = \frac{1}{\pi}\int_{\theta=\theta_1}^{\theta=\theta_2} f(\theta)\sin(\theta_n)d\theta = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} f(\theta)\sin(n\theta)d\theta$$
$$= \frac{1}{\pi}\left[\int_{\theta=0}^{\theta=\pi} f(\theta)\sin(n\theta)d\theta + \int_{\theta=\pi}^{\theta=2\pi} f(\theta)\sin(n\theta)d\theta\right]$$
$$= \frac{1}{\pi}\left[\int_{\theta=0}^{\theta=\pi}\delta(\theta-\tfrac{\pi}{2})\sin(n\theta)d\theta - \int_{\theta=\pi}^{\theta=2\pi}\delta(\theta-\tfrac{3\pi}{2})\sin(n\theta)d\theta\right] = \frac{1}{\pi}\left[\sin(\tfrac{n\pi}{2}) - \sin(\tfrac{3n\pi}{2})\right]$$

Now:

$\cos(n\pi/2) = \cos(3n\pi/2) = 0$ for <u>*odd*</u> $n$ = 1, 3, 5, 7, .... etc.

However:

$\cos(n\pi/2) = \cos(3n\pi/2) = -1$ for <u>*even*</u> $n$ = 2, 6, 10, 14, .... etc.

but:

$\cos(n\pi/2) = \cos(3n\pi/2) = +1$ for <u>*even*</u> $n$ = 4, 8, 12, 16, .... etc.

Thus, for *all* integers, $n > 0$, the Fourier coefficients, $a_n = 0$.

Now:

$\sin(n\pi/2) = -\sin(3n\pi/2) = +1$ for <u>*odd*</u> $n$ = 1, 5, 9, 13, .... etc.

but:

$\sin(n\pi/2) = -\sin(3n\pi/2) = -1$ for <u>*odd*</u> $n$ = 3, 7,11, 15, .... etc.

However:

$\sin(n\pi/2) = \sin(3n\pi/2) = 0$ for <u>*even*</u> $n$ = 2, 4, 6, 8, .... etc.

Thus, only the <u>*odd*</u> Fourier coefficients, $b_n$ are non-zero:

For *odd* $n$ = 1, 5, 9, 13, .... etc., $b_n = 2/\pi$. For *odd* $n$ = 3, 7,11, 15, .... etc., $b_n = -2/\pi$.