Fourier Analysis II:

Some Examples of the Use of Fourier Analysis

A. Fourier Analysis of a Pure-Tone/Single Frequency Waveform

The simplest example of the use of Fourier analysis is that of determining the harmonic content of a pure tone, periodic waveform of a single frequency, f e.g. applied as the input stimulus to a system:

$$S_i(t) = A_i \cos(2\pi f\, t) = A_i \cos(\omega\, t)$$

where A_i is the *amplitude* of the input stimulus, $S_i(t)$ and $\omega = 2\pi f$ is the "angular" frequency, in units of *radians* per second. The *period*, τ of the waveform is $\tau = 1/f$, in units of seconds.

Then if the Fourier series representation of $S_i(t)$ is given by:

$$S_i(t) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos(n\omega t) + \sum_{n=1}^{n=\infty} b_n \sin(n\omega t) = A_i \cos(\omega t)$$

we see *by inspection* that for this equality to hold, the $n = 0$ coefficient, $a_o = 0$, and *all* of the $n > 0$ coefficients, a_n and b_n must also vanish, *except* for the a_1 coefficient, which must be $a_1 = A_i$. Note that these results can also be obtained by explicitly carrying out the inner products $< S_i(t), 1>$, $< S_i(t), \cos(n\omega t)>$ and $< S_i(t), \sin(n\omega t)>$, as defined above.