

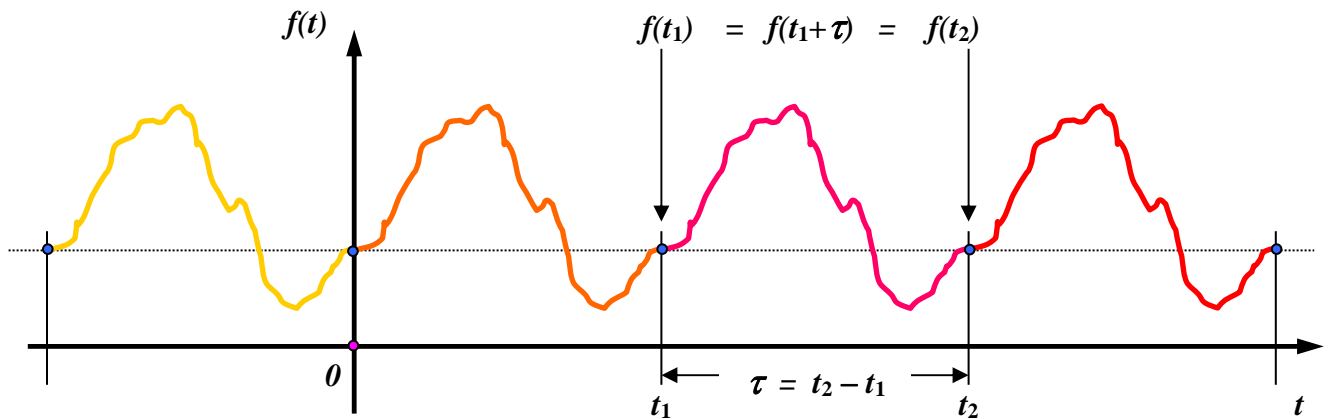
Note that we can also write the Fourier series expansion of $f(x)$ in the time-domain, simply by changing the variable $x \rightarrow t$ and changing the spatial period, L to the temporal (i.e. time) period, τ , i.e. $L \rightarrow \tau$. Then since the frequency, $f = 1/\tau$, and $\omega = 2\pi f$, also with the relation $\omega/k = v$, we have:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos\left(\frac{2\pi n t}{\tau}\right) + \sum_{n=1}^{n=\infty} b_n \sin\left(\frac{2\pi n t}{\tau}\right)$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos(2\pi n f t) + \sum_{n=1}^{n=\infty} b_n \sin(2\pi n f t)$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos(n\omega t) + \sum_{n=1}^{n=\infty} b_n \sin(n\omega t)$$

In the time-domain, the corresponding figure for the periodic temporal function, $f(t)$ is:



Note further that since the *sine* and *cosine* functions, $\sin(x)$ and $\cos(x)$, respectively, are linear combinations of powers of x , (i.e. their Taylor series expansions), then together with 1, they encompass all powers of x . Since the x^n form a complete set of basis vectors for the function “space” associated with the interval $x_1 \leq x \leq x_2$, then 1, and the Taylor series expansions for $\sin(x)$ and $\cos(x)$ also form a complete set of basis vectors for the function “space” associated with the interval $x_1 \leq x \leq x_2$. This is the reason that any mathematically well-behaved, periodic function, $f(x)$ can be precisely replicated by an appropriate linear combination of 1, $\sin(nkx)$ and $\cos(nkx)$ - i.e. a Fourier series expansion, as defined above.