We see that the Fourier coefficients, a_n and b_n are the *real* ("in-phase") and *imaginary* ("90^o out-of-phase") components of the n^{th} *complex* harmonic amplitude, r_n, respectively. The Fourier coefficients, $a_n = |r_n| \cos \delta_n = |r_n| \sin \delta_n'$ and $b_n = |r_n| \sin \delta_n = |r_n| \cos \delta_n'$, where the *magnitude* of the the n^{th} complex harmonic amplitude is $|r_n| = (a_n^2 + b_n^2)^{½}$.

We also see that δ_n and δ_n' are *complementary* phase angles associated with the n^{th} harmonic, since they are related to each other by $\delta_n' = \pi/2 - \delta_n = 90^o - \delta_n$. Note also that the phase angles, δ_n (δ_n') are referenced to the real (imaginary) axes of the complex plane, respectively. By convention, usually we are most interested in the phase angle, δ_n.

Exercises:

1. Work your way through the mathematical details of changing over from the representation(s) of the Fourier series in the space-domain, to those in the time-domain.

2. Work your way through the mathematical details of obtaining the Fourier coefficients, a_0, a_n and b_n from their inner products, in the time-domain.

3. Prove, using the Taylor series expansions for e^x, $\sin(x)$ and $\cos(x)$ that $e^{+i\theta_n} = \cos \theta_n + i \sin \theta_n$ and $e^{-i\theta_n} = \cos \theta_n - i \sin \theta_n$, where $i \equiv \sqrt{(-1)}$, thus $i * i = -1$, and $i * -i = +1$.

4. Work your way through the mathematical details of obtaining the *complex* Fourier series expansion(s) with the c_n & c_{-n} Fourier coefficients, from that with the a_0, a_n and b_n Fourier coefficients.

5. Work your way through the mathematical details of deriving
$$f(\theta) = \frac{|r_0|}{2} + \sum_{n=1}^{n=\infty} |r_n| \cos(\theta_n - \delta_n)$$
from the Fourier series expansion with the the a_0, a_n and b_n Fourier coefficients.

References for Fourier Analysis and Further Reading:

1. Fourier Series and Boundary Value Problems, 2nd Edition, Ruel V. Churchill, McGraw-Hill Book Company, 1969.

2. Mathematics of Classical and Quantum Physics, Volumes 1 & 2, Frederick W. Byron, Jr. and Robert W. Fuller, Addison-Wesley Publishing Company, 1969.

3. Mathematical Methods of Physics, 2nd Edition, Jon Matthews and R.L. Walker, W.A. Benjamin, Inc., 1964.