Fourier Analysis I:

Determination of the Harmonic Content of a Periodic Waveform

The harmonic content of a *periodic* waveform - one which repeats itself in time or in space, can be obtained using the mathematical formalism known as *Fourier analysis* (also known as *harmonic analysis*), named after the French mathematician, Joseph Fourier (1768-1830). The periodic waveform(s) analyzed using this method could be e.g. either a poly-phonic input stimulus to a given system, and/or the linear or non-linear output response waveform associated with that system. Another example of the use of Fourier analysis is to determine the harmonic distortion content and/or the intermodulation distortion content associated with the non-linear response of a system, to which a pure-tone input stimulus is applied.

Mathematically, any arbitrary function, *f(x)* that is *finite*, *single-valued* and *piece-wise continuous* over the interval $x_1 \le x \le x_2$, can be exactly represented by a power series (with suitably-chosen values of the constant coefficients, a_n), due to the fact that the powers of *x*, x^n form a complete set of basis vectors for the function "*space*" associated with the interval $x_1 \le x \le x_2$:

$$f(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + ... = \sum_{n=0}^{n=\infty} a_n x^n$$

In this abstract, infinite-dimensional mathematical space, each of the x^n, as basis vectors, are analogous to the *x*, *y* and *z* axes in real, 3-dimensional space. Except that the complete set of basis vectors, x^n aren't all mutually perpendicular (i.e. *orthogonal*) to each other, like the the *x*, *y* and *z* axes are to each other, in our real, 3-dimensional space. However, *certain linear combinations* of the complete set of x^n are orthogonal to each other. Thus, these certain linear combinations of the x^n in this abstract, infinite-dimensional mathematical space *do* behave exactly analogously to the *x*, *y* and *z* axes in our real, 3-dimensional space. Also, just as one can carry out an infinitude of possible rotations in our real, 3-dimensional space, to obtain a entirely new sets of *x*, *y* and *z* axes in our real, 3-dimensional space, obtaining new *x′*, *y′* and *z′* axes (which are linear combinations of the original *x*, *y* and *z* axes), one can also carry out analogous rotations in the abstract, infinite-dimensional mathematical space, to obtain new complete sets of othogonal basis vectors there, too.

Now, the *sine* and *cosine* functions, *sin (x)* and *cos (x)* have Taylor series expansions in powers of *x* - i.e. the *sin (x)* and *cos (x)* functions are certain specific linear combinations of the x^n:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... = \sum_{n=1}^{n=\infty} \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!}$$

and:

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... = \sum_{n=1}^{n=\infty} \frac{(-1)^{n-1} x^{2n-2}}{(2n-2)!}$$