A Special/Limiting Case – Amplitude Modulation:

When $A_{10} >> A_{20}$ and $f_1 >> f_2$, then the *exact* expression for the total amplitude,

$$A_{tot}(t) = \sqrt{A_1^2(t) + A_2^2(t) + 2A_1(t)A_2(t)\cos\left((\omega_1(t) - \omega_2(t))t + \Delta\varphi_{21}(t)\right)}$$
$$= \sqrt{\begin{array}{l} A_{10}^2\cos^2\left(\omega_1(t)t\right) + A_{20}^2\cos^2\left(\omega_2(t)t\right) \\ +2A_{10}A_{20}\cos\left(\omega_1(t)t\right)\cos\left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)\cos\left((\omega_2(t) - \omega_1(t))t + \Delta\varphi_{21}(t)\right) \end{array}}$$

can be *approximated* by the following expression(s), neglecting terms of order $m^2 \equiv (A_{20}/A_{10})^2 << 1$ under the radical sign, and, defining $\Delta\omega_{21}(t) \equiv (\omega_1(t) - \omega_2(t))$, and noting that for $f_1 >> f_2$, $\Delta\omega_{21}(t) \equiv (\omega_1(t) - \omega_2(t)) \cong \omega_1(t)$:

$$A_{tot}(t) = A_{10}\sqrt{\begin{array}{l} \cos^2\left(\omega_1(t)t\right) + \left(A_{20}/A_{10}\right)^2\cos^2\left(\omega_2(t)t\right) \\ +2\left(A_{20}/A_{10}\right)\cos\left(\omega_1(t)t\right)\cos\left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)\cos\left((\omega_2(t) - \omega_1(t))t + \Delta\varphi_{21}(t)\right) \end{array}}$$
$$\simeq A_{10}\sqrt{\cos^2\left(\omega_1(t)t\right) + 2\left(A_{20}/A_{10}\right)\cos\left(\omega_1(t)t\right)\cos\left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)\cos\left(\Delta\omega_{21}(t)t + \Delta\varphi_{21}(t)\right)}$$
$$\simeq A_{10}\sqrt{\cos^2\left(\omega_1(t)t\right) + 2\left(A_{20}/A_{10}\right)\cos\left(\omega_1(t)t\right)\cos\left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)\cos\left(\omega_1(t)t + \Delta\varphi_{21}(t)\right)}$$

However, when $f_1 >> f_2$, the relative phase difference $\Delta\varphi_{21}(t)$ changes by 2π radians (essentially) for *each* cycle of the frequency, f_1. Hence we can safely set to zero this phase difference, i.e. $\Delta\varphi_{21}(t) = 0$ because its (time-averaged) effect in the $f_1 >> f_2$ limit is negligible. Using the Taylor series expansion $\sqrt{1+\varepsilon} \simeq 1 + \frac{1}{2}\varepsilon$ for the case when $\varepsilon << 1$, the expression for the total amplitude then becomes:

$$A_{tot}(t) \simeq A_{10}\sqrt{\cos^2\left(\omega_1(t)t\right) + 2\left(A_{20}/A_{10}\right)\cos\left(\omega_1(t)t\right)\cos\left(\omega_2(t)t\right)\cos\left(\omega_1(t)t\right)}$$
$$\simeq A_{10}\sqrt{\cos^2\left(\omega_1(t)t\right) + 2\left(A_{20}/A_{10}\right)\cos^2\left(\omega_1(t)t\right)\cos\left(\omega_2(t)t\right)}$$
$$\simeq A_{10}\cos\left(\omega_1(t)t\right)\sqrt{1 + 2\left(A_{20}/A_{10}\right)\cos\left(\omega_2(t)t\right)}$$
$$\simeq A_{10}\cos\left(\omega_1(t)t\right)\left(1 + \left(A_{20}/A_{10}\right)\cos\left(\omega_2(t)t\right)\right)$$
$$\simeq A_{10}\cos\left(\omega_1(t)t\right)\left(1 + m\cos\left(\omega_2(t)t\right)\right)$$

The ratio $m \equiv (A_{20}/A_{10}) << 1$ is known as the (amplitude) *modulation depth* of the high-frequency *carrier* wave $A_1(t)$, with amplitude $A_{10} >> A_{20}$ and frequency $f_1 >> f_2$, *modulated* by the low frequency wave $A_2(t)$, with amplitude A_{20} and frequency f_2. This is the underlying principle of *AM radio* – *AM* stands for ***A**mplitude **M**odulation…*

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