

The primary reason that the phenomenon of beats is discussed more often in terms of intensity, rather than amplitude is that the physics is perhaps easier to understand from the intensity perspective – at least mathematically, things appear more obvious, physically:

$$I_{tot}(t) = |A_{tot}(t)|^2 = |A_1(t) + A_2(t)|^2$$

$$I_{tot}(t) = A_{10}^2 \cos^2(\omega_1(t)t) + A_{20}^2 \cos^2(\omega_2(t)t + \Delta\phi_{21}(t)) + 2A_{10}A_{20} \cos(\omega_1(t)t) \cos(\omega_2(t)t + \Delta\phi_{21}(t))$$

Let us define:

$$\mathcal{G}_1(t) \equiv \omega_1(t)t \qquad \mathcal{G}_2(t) \equiv (\omega_2(t)t + \Delta\phi_{21}(t))$$

And then let us use the mathematical identity:

$$\cos \mathcal{G}_1 \cos \mathcal{G}_2 \equiv \frac{1}{2} [\cos(\mathcal{G}_2 + \mathcal{G}_1) + \cos(\mathcal{G}_2 - \mathcal{G}_1)]$$

Thus:

$$\begin{aligned} I_{tot}(t) &= A_{10}^2 \cos^2(\omega_1(t)t) + A_{20}^2 \cos^2(\omega_2(t)t + \Delta\phi_{21}(t)) \\ &+ A_{10}A_{20} [\cos((\omega_2(t) + \omega_1(t))t + \Delta\phi_{21}(t)) + \cos((\omega_2 - \omega_1)t - \Delta\phi_{21}(t))] \end{aligned}$$

The let us define:

$$\Omega_{21}(t) \equiv (\omega_2(t) + \omega_1(t)) \qquad \Delta\omega_{21}(t) \equiv |\omega_2(t) - \omega_1(t)|$$

We then obtain:

$$I_{tot}(t) = A_{10}^2 \cos^2(\omega_1(t)t) + A_{20}^2 \cos^2(\omega_2(t)t + \Delta\phi_{21}(t)) + A_{10}A_{20} [\cos(\Omega_{21}(t)t + \Delta\phi_{21}(t)) + \cos(\Delta\omega_{21}(t)t - \Delta\phi_{21}(t))]$$

Using the identity:

$$\cos^2 \mathcal{G} = \cos \mathcal{G} \cos \mathcal{G} \equiv \frac{1}{2} [\cos 0 + \cos 2\mathcal{G}] = \frac{1}{2} [1 + \cos 2\mathcal{G}]$$

We then obtain the additional relation, which is not usually presented and/or discussed in physics textbooks:

$$\begin{aligned} I_{tot}(t) &= \frac{1}{2} A_{10}^2 [1 + \cos^2 2(\omega_1(t)t)] + \frac{1}{2} A_{20}^2 [1 + \cos^2 2(\omega_2(t)t + \Delta\phi_{21}(t))] \\ &+ A_{10}A_{20} [\cos(\Omega_{21}(t)t + \Delta\phi_{21}(t)) + \cos(\Delta\omega_{21}(t)t - \Delta\phi_{21}(t))] \end{aligned}$$

This latter formula shows that there are a.) DC (i.e. zero frequency) components/constant terms associated with both amplitudes, A_{10} and A_{20} , b.) 2nd harmonic components with $2f_1$ and $2f_2$, as well as c.) a component associated with the sum of the two frequencies, $\Omega_{21} = f_1 + f_2$, and d.) a component associated with the difference of the two frequencies, $\Delta f_{21} = f_1 - f_2$. This is a remarkably similar result to that associated with the output response from a system with a quadratic non-linear response to a pure/single-frequency sine-wave input!