In terms of the phasor diagram, as time progresses, the individual amplitudes  $A_1(t)$  and  $A_2(t)$  actually precess at (angular) rates of  $\omega_1 = 2\pi f_1$  and  $\omega_2 = 2\pi f_2$  radians per second respectively, completing one revolution in the phasor diagram, for each cycle/each period of  $\tau_1 = 2\pi/\omega_1 = 1/f_1$  and  $\tau_2 = 2\pi/\omega_2 = 1/f_2$ , respectively. If at time t = 0 the two phasors are precisely in phase with each other (i.e. with initial relative phase  $\Delta \phi_{21} = 0.0$ ), then the resultant/total amplitude,  $A_{tot}(t = 0) = A_1(t = 0) + A_2(t = 0)$  will be as shown in the figure below.

$$A_{tot}(t=0) = A_1(t=0) + A_2(t=0)$$
  
 $A_1(t=0) \qquad A_2(t=0)$ 

As time progresses, if  $\omega_1 \neq \omega_2$ , (phasor 1 with angular frequency  $\omega_1 = 2\pi f_1 = 2*1000\pi = 2000\pi$  radians/sec and  $\omega_2 = 2\pi f_2 = 2*980\pi = 1960\pi$  radians/sec in our example above) phasor 1, with higher angular frequency will precess more rapidly than phasor 2 (by the difference in angular frequencies,  $\Delta \omega = (\omega_1 - \omega_2) = (2000\pi - 1960\pi) = 40\pi$  radians/second). Thus, as time increases, phasor 1 will <u>lead</u> phasor 2; eventually (at time  $t = \frac{1}{2}\tau_{beat} = 0.025 = 1/40^{th}$  sec in our above example) phasor 2 will be exactly  $\Delta \varphi = \pi$  radians, or 180 degrees behind in phase relative to phasor 1. Phasor 1 at time  $t = \frac{1}{2}\tau_{beat} = 0.025$  sec  $= 1/40^{th}$  sec will be oriented exactly as it was at time t = 0.0 (having precessed exactly  $N_1 = \omega_1 t/2\pi = 2\pi f_1 t/2\pi = f_1 t = 25.0$  revolutions in this time period), however phasor 2 will be pointing in the opposite direction at this instant in time (having precessed only  $N_2 = \omega_2 t/2\pi = 2\pi f_2 t/2\pi = f_2 t = 24.5$  revolutions in this same time period), and thus the total amplitude  $A_{tot}(t = \frac{1}{2}\tau_{beat}) = A_1(t = \frac{1}{2}\tau_{beat}) + A_2(t = \frac{1}{2}\tau_{beat})$  will be zero (if the magnitudes of the two individual amplitudes are precisely equal to each other), or minimal (if the magnitudes of the two individual amplitudes are not precisely equal to each other), as shown in the figure below.

$$A_{tot}(t = \frac{1}{2}\tau_{beat}) = A_1(t = \frac{1}{2}\tau_{beat}) + A_2(t = \frac{1}{2}\tau_{beat}) = 0$$

$$A_2(t = \frac{1}{2}\tau_{beat}) = -A_1(t = \frac{1}{2}\tau_{beat})$$

As time progresses further, phasor 2 will continue to lag farther and farther behind, and eventually (at time  $t = \tau_{beat} = 0.050 \text{ sec} = 1/20^{\text{th}}$  sec in our above example) phasor 2, having precessed through N<sub>2</sub> = 49.0 revolutions will now be exactly  $\Delta \phi = 2\pi$  radians, or 360 degrees (or one full revolution) behind in phase relative to phasor 1 (which has precessed through N<sub>1</sub> = 50.0 full revolutions), thus, the net/overall result is the same as being exactly in phase with phasor 1! At this point in time, A<sub>tot</sub>(t =  $\tau_{beat}$ ) = A<sub>1</sub>(t =  $\tau_{beat}$ ) + A<sub>2</sub>(t =  $\tau_{beat}$ ) = 2A<sub>1</sub>(t =  $\tau_{beat}$ ) = 2A<sub>1</sub>(t =  $\tau_{beat}$ ), and the phasor diagram looks precisely like that at time t = 0.

Thus, it should (hopefully) now be clear to the reader that <u>the phenomenon of beats is</u> <u>manifestly that of time-dependent alternating constructive/destructive interference between two</u> <u>periodic signals of comparable frequency, at the amplitude level</u>. This is by no means a trivial point, as often the beats phenomenon is discussed in physics textbooks in the context of intensity,  $I_{tot}(t) = |A_{tot}(t)|^2 = |A_1(t) + A_2(t)|^2$ . From the above discussion, the <u>physics origin</u> of the beats phenomenon has absolutely <u>nothing</u> to do with intensity of the overall/ resultant signal.

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