

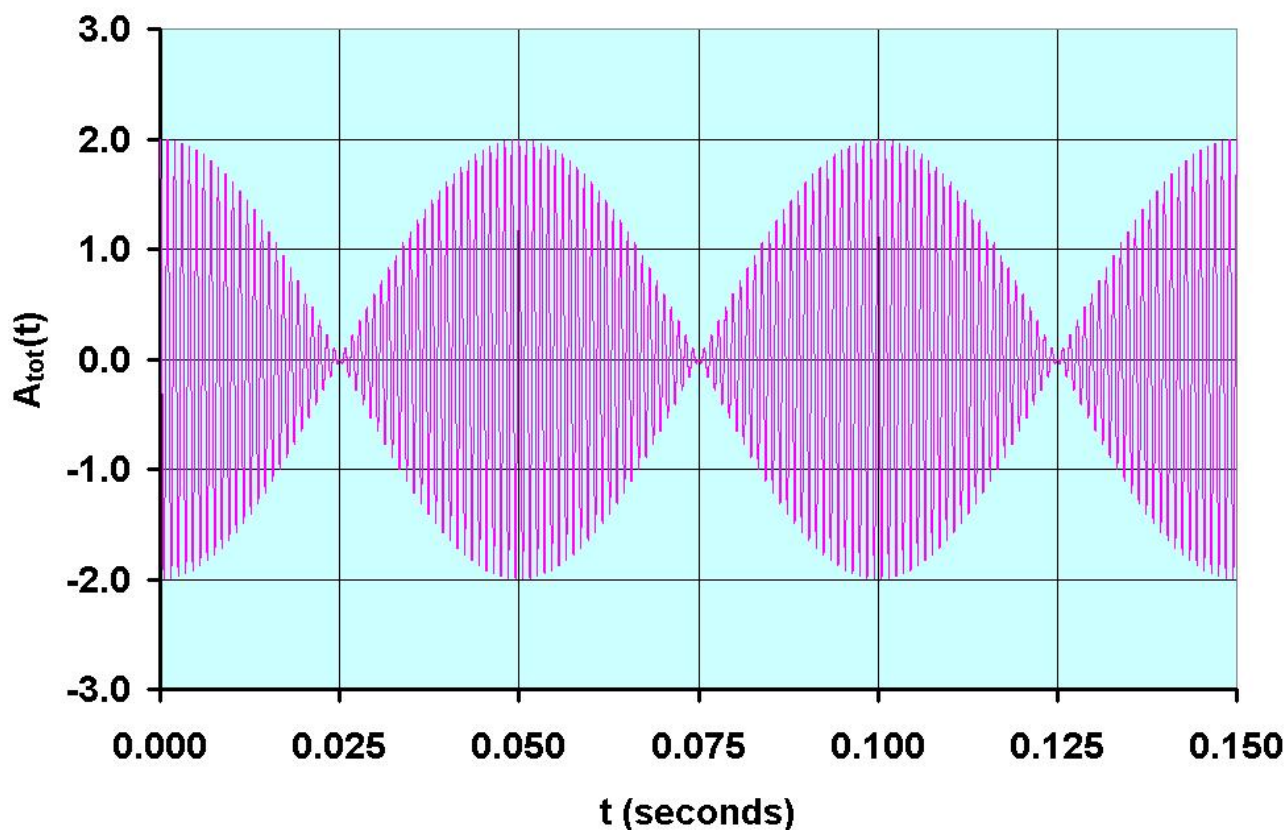
The phase of the total amplitude, $A_{\text{tot}}(t)$ relative to that of the first amplitude $A_1(t)$, at an arbitrary time, t is $\Delta\psi(t)$ and is obtained from the projections of the total amplitude phasor, $A_{\text{tot}}(t)$ onto the y- and x- axes of the 2-D phasor plane:

$$\tan\psi = \frac{A_2(t)\cos(\omega_2(t)t + \Delta\phi_{21}(t))\sin\Delta\phi_{21}(t)}{A_1(t)\cos(\omega_1(t)t) + A_2(t)\cos(\omega_2(t)t + \Delta\phi_{21}(t))\cos\Delta\phi_{21}(t)}$$

The total amplitude, $A_{\text{tot}}(t) = A_1(t) + A_2(t)$ vs. time, t is shown in the figure below, for time-independent/constant frequencies of $f_1 = 1000$ Hz and $f_2 = 980$ Hz, equal amplitudes of unit strength, $A_{10} = A_{20} = 1.0$ and zero relative phase, $\Delta\phi_{21} = 0.0$

Beats Phenomenon

$$A_{\text{tot}}(t) = A_1(t) + A_2(t)$$



Clearly, the beats phenomenon can be seen in the above waveform of total amplitude, $A_{\text{tot}}(t) = A_1(t) + A_2(t)$ vs. time, t . From the above graph, it is obvious that the beat period, $\tau_{\text{beat}} = 1/f_{\text{beat}} = 0.050$ sec = $1/20^{\text{th}}$ sec, corresponding to a beat frequency, $f_{\text{beat}} = 1/\tau_{\text{beat}} = 20$ Hz, which is simply the frequency difference, $f_{\text{beat}} \equiv |f_1 - f_2|$ between $f_1 = 1000$ Hz and $f_2 = 980$ Hz. Thus, the beat period, $\tau_{\text{beat}} = 1/f_{\text{beat}} = 1/|f_1 - f_2|$. When $f_1 = f_2$, the beat period becomes infinitely long, and no beats are heard.