The same mathematical formalism can be used for adding together two arbitrary (but still periodic) signals, $Z_1(t) = A_1(t)\exp\{i(\omega_1(t)t + \varphi_1(t))\}$ and $Z_2(t) = A_2(t)\exp\{i(\omega_2(t)t + \varphi_2(t))\}$. The individual amplitudes, frequencies and phases may be time-dependent. The resultant overall complex amplitude is

$$Z_{tot}(t) = Z_1(t) + Z_2(t) = A_1(t)\exp\{i(\omega_1(t)t + \varphi_1(t))\} + A_2(t)\exp\{i(\omega_2(t)t + \varphi_2(t))\}.$$

Because the zero of time is (always) arbitrary, we are free to choose/redefine t = 0 in such a way as to rotate away one of the two phases – absorbing it as an overall/absolute phase (which is physically unobservable). Since $e^{(x+y)} = e^x e^y$, the above formula can be rewritten as:

$$Z_{tot}(t) = Z_1(t) + Z_2(t) = A_1(t)\exp\{i\omega_1(t)t\}\exp\{i\varphi_1(t)\} + A_2(t)\exp\{i\omega_2(t)t\}\exp\{i\varphi_2(t)\}.$$

Multiplying both sides of this equation by $\exp\{-i\varphi_1(t)\}$:

$$Z_{tot}(t)\exp\{-i\varphi_1(t) = A_1(t)\exp\{i\omega_1(t)t\} + A_2(t)\exp\{i\omega_2(t)t\}\exp\{i(\varphi_2(t)-\varphi_1(t))\}.$$

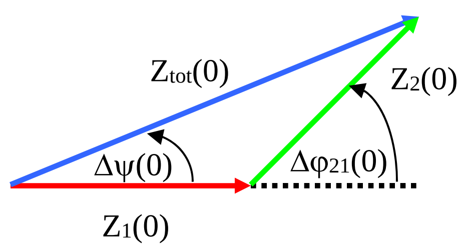
This shift in overall phase, by an amount $\exp\{-i\varphi_1(t)\}$is formally equivalent to a redefinition to the zero of time, and also physically corresponds to a (simultaneous) rotation of both of the (mutually-perpendicular) real and imaginary axes in the complex plane by an angle, $\varphi_1(t)$. The physical meaning of the remaining phase after this redefinition of time/shift in overall phase is a phase *difference* between the second complex amplitude, $Z_2(t)$ relative to the first, $Z_1(t)$ is $\Delta\varphi_{21}(t) \equiv \varphi_2(t) - \varphi_1(t)$. Thus, at the (newly) redefined time $t^* = t - \varphi_1(t)/\omega_1(t) = 0$ (and then substituting $t^* \Rightarrow t$) the resulting overall, time-redefined amplitude is:

$$Z_{tot}(t) = A_1(t)\exp\{i\omega_1(t)t\} + A_2(t)\exp\{i\omega_2(t)t\}\exp\{i\Delta\varphi_{21}(t)\}$$

or:

$$Z_{tot}(t) = A_1(t)\exp\{i\omega_1(t)t\} + A_2(t)\exp\{i(\omega_2(t)t + \Delta\varphi_{21}(t))\}.$$

The so-called *phasor relation* for the two individual complex amplitudes, $Z_1(t)$, $Z_2(t)$ and the resulting overall amplitude, $Z_{tot}(t)$ in the complex plane is shown in the figure below.



The magnitude of the resulting overall amplitude, $|Z_{tot}(t)|$ can be obtained from

$$\begin{aligned}
|Z_{tot}(t)|^2 &= Z_{tot}(t)Z^*_{tot}(t)\\
&= |Z_1(t) + Z_2(t)|^2 = (Z_1(t) + Z_2(t))(Z_1(t) + Z_2(t))^* = (Z_1(t) + Z_2(t))(Z^*_1(t) + Z^*_2(t))\\
&= Z_1(t)Z^*_1(t) + Z_2(t)Z^*_1(t) + Z_1(t)Z^*_2(t) + Z_2(t)Z^*_2(t)\\
&= |Z_1(t)|^2 + |Z_2(t)|^2 + 2\mathrm{Re}[Z_1(t)Z^*_2(t)]\\
&= A_1^2(t) + A_2^2(t) + 2A_1(t)A_2(t)\mathrm{Re}[\exp\{-i((\omega_2(t)-\omega_1(t))t + \Delta\varphi_{21}(t))\}]\\
&= A_1^2(t) + A_2^2(t) + 2A_1(t)A_2(t)\mathrm{Cos}\{(\omega_2(t)-\omega_1(t))t + \Delta\varphi_{21}(t)\}
\end{aligned}$$