Acoustical Interference Phenomena

When two (or more) periodic signals are linearly superposed (*i.e.* added together), the resultant/overall waveform that results depends on the amplitude, frequency and phase information associated with the individual signals. Mathematically, this is most easily and transparently described using complex notation.

Basics of/Primer on Complex Variables

Complex variables are used whenever phase information is important. A complex function, $Z = X+iY$ consists of two portions, a so-called "real" part of Z, denoted $X = Re(Z)$ and a so-called "imaginary" part of Z, denoted $Y = Im(Z)$. The number $i \equiv \sqrt{(-1)}$. The magnitude of the complex variable, Z is designated as $|Z| = \sqrt{(Z)^2}$, $= \sqrt{(ZZ^*)}$, with $Z^2 \equiv ZZ^*$ where Z^* is the so-called complex conjugate of Z, i.e. $Z^* = (Z)^* = (X+iY)^* = X–iY$, with $i^* = (i)^* \equiv –\sqrt{(-1)}$. Thus, $|Z| = \sqrt{(ZZ^*)} = \sqrt{(X+iY)(X+iY)^*} = \sqrt{(X+iY)(X–iY)} = \sqrt{(X^2 + iXY – iXY + Y^2)} = \sqrt{(X^2+Y^2)}$. Thus the magnitude of Z, $|Z|$ is analogous to the hypotenuse, c of a right triangle ($c^2 = a^2 + b^2$) and/or the radius of a circle, r centered at the origin ($r^2 = x^2+ y^2$).

Because complex variables $Z = X+iY$ consist of two components, Z can be graphically depicted as a 2-component "vector" $Z = (X,Y)$ lying in the so-called complex plane, as shown in the figure below. The real component of Z, $X = Re(Z)$ is by convention drawn along the x, or horizontal axis (i.e. the abscissa). The imaginary component of Z, $Y = Im(Z)$ is by convention drawn along the y, or vertical axis (i.e. the ordinate), as shown in the figure below.