Supplement: Wave Uncertainty
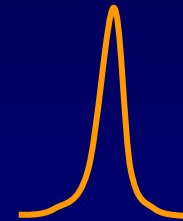
Mathematically, one can produce a localized function by superposing sine waves with a "spread" of wave numbers, Δk: $\Delta k \cdot \Delta x \geq 1$.

This is a result of Fourier analysis, which most of you will learn in Math.

It means that making a short wave packet requires a broad spread in wavelengths.
Conversely, a single-wavelength wave would extend forever.

So far, this is just math. The physics comes in when we make the wavelength-momentum connection: $p = h/\lambda = \hbar k$.

Example:
How many of you have experienced a close lightning strike (within a couple hundred feet)? If you were paying attention, you may have noticed that the sound, which is a very short pulse, is very weird. That weirdness is a result of the very broad range of frequencies that is needed to construct a very short pulse. One doesn't normally experience such a broad frequency range.