

Supplement: Phase shift and Position or Time Shift

Because the wave is oscillating both in time and position, we can consider ϕ to be either a time or position shift:

Time:

$$\begin{aligned}y &= A_1 \cos(kx - \omega t + \phi) \\ &= A_1 \cos(kx - \omega(t - \phi/\omega)) \\ &= A_1 \cos(kx - \omega(t - \phi T/2\pi)) \\ &= A_1 \cos(kx - \omega(t - \delta t))\end{aligned}$$

The time shift: $\delta t/T = \phi/2\pi$

Positive ϕ shifts to later times.

Position:

$$\begin{aligned}y &= A_1 \cos(kx - \omega t + \phi) \\ &= A_1 \cos(k(x + \phi/k) - \omega t) \\ &= A_1 \cos(k(x + \phi\lambda/2\pi) - \omega t) \\ &= A_1 \cos(k(x - \delta x) - \omega t)\end{aligned}$$

The position shift: $\delta x/\lambda = -\phi/2\pi$

Positive ϕ shifts to negative position.