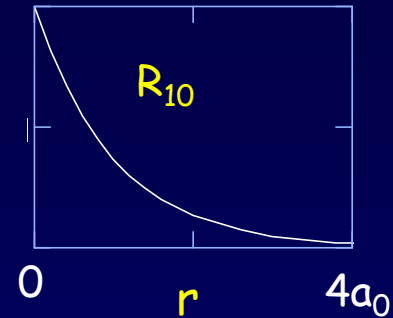


Solution

What is the normalization constant for the hydrogen atom ground state?

$$\psi_{100}(r, \theta, \phi) = NR_{10}(r) = Ne^{-r/a_0}$$



The probability density is $|\psi|^2 = N^2 \exp(-2r/a_0)$.
In 3D, this means “probability per unit volume”.

We require that the total probability = 1: $\int |\psi|^2 dV = 1$

$$dV = r^2 \sin\theta dr d\theta d\phi$$

With spherical symmetry, the angular integrals give 4π , so we are left with:

$$4\pi N^2 \int_0^{\infty} r^2 e^{-2r/a_0} dr = 1 \Rightarrow N^2 = \frac{1}{\pi a_0^3}$$

“You can look it up!”

$$\psi_{100}(r) = \sqrt{\frac{1}{\pi a_0^3}} e^{-r/a_0}$$

Normalized ground-state wave function of hydrogen