Solution

Consider an electron around a nucleus that has two protons, (an ionized Helium atom).

1. Compare the "effective Bohr radius" $a_{0,He}$ with the usual Bohr radius for hydrogen, a_0:

   a. $a_{0,He} > a_0$

   b. $a_{0,He} = a_0$

   c. $a_{0,He} < a_0$

Look at how a_0 depends on the charge:

$$a_0 \equiv \frac{\hbar^2}{m\kappa e^2} \Rightarrow a_{0,He} \equiv \frac{\hbar^2}{m\kappa(2e)e} = \frac{a_0}{2}$$

This should make sense:
more charge → stronger attraction
→ electron "sits" closer to the nucleus

2. What is the ratio of ground state energies $E_{0,He}/E_{0,H}$?

   a. $E_{0,He}/E_{0,H} = 1$

   b. $E_{0,He}/E_{0,H} = 2$

   c. $E_{0,He}/E_{0,H} = 4$

Clearly the electron will be more tightly bound, so $|E_{0,He}| > |E_{0,H}|$. How much more tightly? Look at E_0:

$$E_{0,H} = -\frac{m\kappa^2 e^4}{2\hbar^2} \quad \Rightarrow \quad E_{0,He} = \frac{-m\kappa^2 (2e)^2 e^2}{2\hbar^2} = 4E_{0,H}$$

In general, for a "hydrogenic" atom (only one electron) with Z protons:

$$E_{0,Z} = Z^2 E_{0,H}$$