Angular Momentum & Uncertainty Principle

Note that $L^2 = l(l+1)\hbar^2$ not $(l\hbar)^2$
Also, we describe angular momentum using only two numbers, l and m.

Q: Why can't we specify all three components (*e.g.*, L =(0,0,l) so that $L^2 = l^2$?

A: The uncertainty principle doesn't allow us to know that both $L_x = 0$ and $L_y = 0$ unless $L_z = 0$ also.

Proof by contradiction: Assume L =(0,0,l).

$\vec{L} = \vec{r} \times \vec{p}$, so if *L* points along the z-axis, both *r* and *p* lie in the x-y plane.
This means that $\Delta z = 0$ and $\Delta p_z = 0$, violating the uncertainty principle.

Thus, L must have a nonzero L_x or L_y, making L^2 somewhat larger.
We can't specify all three components of the angular momentum vector.

This logic only works for $L \neq 0$. L = (0,0,0) is allowed. It's the s-state.

All physical quantities are subject to uncertainty relations, not just position and momentum.