Quantized Angular Momentum

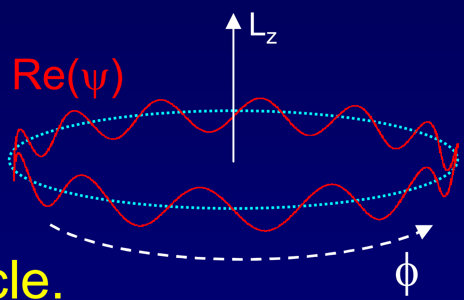
Linear momentum depends on the wavelength ($k=2\pi/\lambda$):

$$p = \hbar k \text{ where } \psi(x) \propto e^{ikx}$$

Angular momentum depends on the tangential component of the momentum. Therefore L_z depends on the wavelength as one moves around a circle in the x-y plane. Therefore, a state with L_z has a similar form:

$$L_z = m\hbar \text{ where } \psi(\vec{r}) \propto Y_{lm}(\theta,\phi) \propto e^{im\phi}$$

We're ignoring R(r) for now.



An important boundary condition:
An integer number of wavelengths must fit around the circle.
Otherwise, the wave function is not single-valued.

Reminder:
$e^{im\phi} = \cos(m\phi) + i\sin(m\phi)$

This implies that $m = 0, \pm1, \pm2, \pm3, \ldots$

and $L_z = 0, \pm\hbar, \pm2\hbar, \pm3\hbar, \ldots$

Angular momentum is quantized!!

http://www.falstad.com/qmatom/