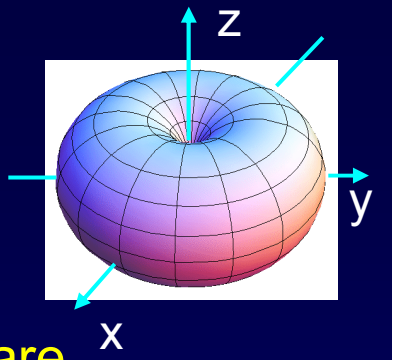
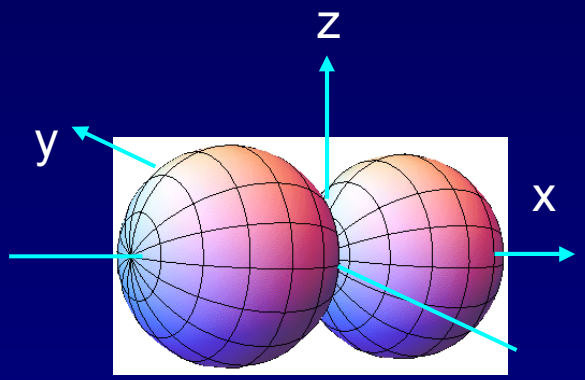
Cylindrical Symmetry



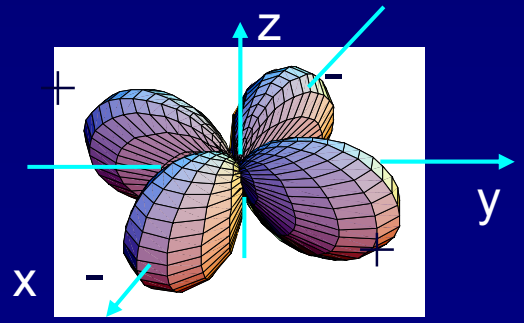
Why do none of the graphs display ϕ-dependence?
(They all have cylindrical symmetry.)

For a given m, the ϕ dependence of ψ is $e^{im\phi}$. When we square it to find the probability, $e^{im\phi}e^{-im\phi} = 1$.
In order to see ϕ dependence, we need a superposition of different m's.

For example, consider the superposition:
(l = 1, m = +1) & (l = 1, m = -1).
This will have an azimuthal wave function:
$e^{i\phi} + e^{-i\phi} \approx \cos\phi$, i.e., lobes along the x-axis:



Similar arguments explain how to create the usual "d" lobes, from l =2, m = $\pm$2 superpositions:



See Supplement for more info.