Appendix: Solving the 'Radial' SEQ for H --deriving a_0 and E

- Substituting $R(r) = Ne^{-\alpha r}$ into $\left(\frac{-\hbar^2}{2m}\frac{1}{r}\frac{\partial^2}{dr^2}r - \frac{\kappa e^2}{r}\right)R(r) = ER(r)$, we get:

$$\frac{-\hbar^2}{2m}\frac{1}{r}\left(-2\alpha e^{-\alpha r} + \alpha^2 r e^{-\alpha r}\right) - \frac{\kappa e^2}{r}e^{-\alpha r} = Ee^{-\alpha r}$$

- For this equation to hold for all r, we must have:

$$\frac{\hbar^2\alpha}{m} = \kappa e^2 \qquad \text{AND} \qquad \frac{-\hbar^2\alpha^2}{2m} = E$$

$$\alpha = \frac{m\kappa e^2}{\hbar^2} \equiv \frac{1}{a_0} \qquad \Rightarrow \qquad E = \frac{-\hbar^2}{2ma_0^2}$$

- Evaluating the ground state energy:

$$E = \frac{-\hbar^2}{2ma_0^2} = \frac{-\hbar^2c^2}{2mc^2a_0^2} = \frac{-(197)^2}{2(.51)(10^6)(.053)^2} = -13.6\,\text{eV}$$