Supplement: Separation of Variables (2)

Regroup:

$$\left[-\frac{\hbar^2}{2m}\frac{1}{f}\frac{d^2f}{\partial x^2} + U(x) \right] + \left[-\frac{\hbar^2}{2m}\frac{1}{g}\frac{d^2g}{dy^2} + U(y) \right] + \left[-\frac{\hbar^2}{2m}\frac{1}{h}\frac{d^2h}{dz^2} + U(z) \right] = E$$

A function of x   A function of y   A function of z

We have three functions, each depending on a different variable, that must sum to a constant.
Therefore, each function must be a constant:

$$-\frac{\hbar^2}{2m}\frac{1}{f}\frac{d^2f}{\partial x^2} + U(x) = E_x$$

$$-\frac{\hbar^2}{2m}\frac{1}{g}\frac{d^2g}{dy^2} + U(y) = E_y$$

Each function, f(x), g(y), and h(z) satisfies its own 1D SEQ.

$$-\frac{\hbar^2}{2m}\frac{1}{h}\frac{d^2h}{dz^2} + U(z) = E_z$$

$$E_x + E_y + E_z = E$$