Supplement: Separation of Variables (1)

In the 3D box, the SEQ is:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\big(U(x)+U(y)+U(z)\big)\psi=E\psi$$

NOTE:
Partial derivatives.

Let's see if separation of variables works.
Substitute this expression for ψ into the SEQ:

$$\psi(x,y,z)=f(x)g(y)h(z)$$

$$-\frac{\hbar^2}{2m}\left(gh\frac{d^2f}{dx^2}+fh\frac{d^2g}{dy^2}+fg\frac{d^2h}{dz^2}\right)+\big(U(x)+U(y)+U(z)\big)fgh=Efgh$$

NOTE:
Total derivatives.

Divide by fgh:

$$-\frac{\hbar^2}{2m}\left(\frac{1}{f}\frac{d^2f}{\partial x^2}+\frac{1}{g}\frac{d^2g}{dy^2}+\frac{1}{h}\frac{d^2h}{dz^2}\right)+\big(U(x)+U(y)+U(z)\big)=E$$