Supplement:
Time-Energy Uncertainty Principle

Now that we are considering time-dependent problems, it is a good time to introduce another application of the Heisenberg Uncertainty Principle, based on measurements of energy and time. We start from our previous result:

$$\Delta p\,\Delta x \geq \hbar \quad \Rightarrow \quad (c\Delta p)\left(\frac{\Delta x}{c}\right) \geq \hbar \quad \Rightarrow \quad \Delta E\,\Delta t \geq \hbar$$

Sometimes this is further transformed as follows:

$$\Delta E\,\Delta t \geq \hbar \quad \Rightarrow \quad (\hbar\Delta\omega)\,\Delta t \geq \hbar$$

$$\Delta\omega\,\Delta t \geq 1 \quad \Rightarrow \quad \Delta f\,\Delta t \geq 1/2\pi$$

The last line is a standard result from Fourier wave analysis; this should not surprise us – the Uncertainty Principle arises simply because particles behave as waves that are oscillating in time as well as in space.