Normalizing Superpositions

We want the total probability to equal 1,
even when the particle is in a superposition of states:

$$P_{tot} = \int |\psi|^2\, dx = \int |a\psi_1 + b\psi_2|^2\, dx = 1$$

This looks like a mess. However, we're in luck. Multiply it out:

$$\int |a\psi_1 + b\psi_2|^2 dx = \int |a\psi_1|^2 dx + \int |b\psi_2|^2 dx + \int (a\psi_1)^* (b\psi_2) dx + \int (b\psi_2)^* (a\psi_1) dx$$

$$= |a|^2 \int |\psi_1|^2 dx + |b|^2 \int |\psi_2|^2 dx + a^* b \int (\psi_1)^* (\psi_2) dx + b^* a \int (\psi_2)^* (\psi_1) dx$$

$$= |a|^2 + |b|^2 + 0 + 0$$

If ψ_1 is normalized

If ψ_2 is normalized

It is a mathematical theorem that these integrals always = 0 if the energies are different.

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A normalized superposition must have $|a|^2 + |b|^2 = 1$.

$\psi = 0.8\psi_1 + 0.6\psi_2$ is normalized.
$\psi = 0.5\psi_1 + 0.5\psi_2$ is not normalized.