Example: Motion of a Free Particle

A free particle moves without applied forces; so set U(x) = 0.

The SEQ is now reasonably simple:

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x,t)}{dx^2} = i\hbar\frac{d\Psi(x,t)}{dt}$$

The second x-derivative is proportional to the first t-derivative.
Here's one solution:

$$\boxed{\Psi(x,t) = Ae^{i(kx-\omega t)}}$$

This is a traveling wave. The particle has

Momentum: $\boxed{p = \hbar k = h/\lambda}$
Energy: $\boxed{E = \hbar\omega = hf}$

Not a surprise, I hope.

Check it. Take the derivatives:

$$\frac{\partial\Psi}{\partial x} = ik\ Ae^{i(kx-\omega t)}$$

$$\frac{\partial^2\Psi}{\partial x^2} = (ik)^2 Ae^{i(kx-\omega t)} = -k^2 Ae^{i(kx-\omega t)}$$

$$\frac{\partial\Psi}{\partial t} = (-i\omega)Ae^{i(kx-\omega t)}$$

So, it works if: $\boxed{\frac{\hbar^2 k^2}{2m} = \hbar\omega}$

That's the same as: $\boxed{\frac{p^2}{2m} = E}$