Time-dependence of Energy Eigenstates

The time-independent SEQ is just a special case of the time-dependent SEQ.
So, if Ψ(x,t) is a state with definite energy, it is a solution to both equations.

Both equations have the same left-hand side, so the right sides must be equal:

RHS of time ind. SEQ

$$E\Psi(x,t) = i\hbar \frac{d\Psi(x,t)}{dt}$$

RHS of time dep. SEQ

This equation has the solution:

$$\Psi(x,t) = \psi(x)e^{-i\omega t} \quad \text{with } \omega = \frac{E}{\hbar}$$

Notes:

- ψ(x) is not determined. We need the LHS for that.
- E = ħω = hf, as expected.

Ψ is complex. However, we are interested in $|\Psi|^2$, because that's what we measure.

$$|\Psi(x,t)|^2 = \left(\psi^*(x)e^{+i\omega t}\right)\left(\psi(x)e^{-i\omega t}\right) = |\psi(x)|^2$$

This is *always* a real number.

So, for an energy eigenstate:
the probability density, $|\Psi|^2$, has no time dependence! (i.e., it's a "stationary state".)

We don't actually need the time-independent SEQ, but if we know we're dealing with energy eigenstates, the math is simpler.