Another Example

What we have seen here is that if we solve $Ax = 0$, generate the factorization $A = LU$ and find L^{-1}, we can immediately construct bases for all four fundamental subspaces.

Ex:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, L^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

Immediately we see

$$C(A) = \text{ Span}\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}\}, C(A^T) = \text{ Span}\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}\}, N(A^T) = \text{ Span}\{\begin{bmatrix} -3 \\ 1 \end{bmatrix}\}$$

Also we note that $Ax = 0 \Leftrightarrow Ux = 0 \Leftrightarrow u + 2v = 0$ with free variable v. The special solution corresponding to this free variable comes from setting $v = 1$, so $u = -2$. Thus

$$N(A) = \text{ Span}\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}\}$$