

The Column Space

The special solutions are always linearly independent (why?).
Consequently:

The special solutions of $Ax = 0$ form a basis of $N(A)$
 $\dim N(A) = n - r = \text{nullity}(A)$

Consider next the **column space** $C(A)$. Previously we saw

- pivot variable columns are always linearly independent
- free variable columns are linear combos of pivot columns (i.e. each special solution of $Ax = 0$ gives a dependency condition)

Therefore (since we can throw away the free variable columns):

The pivot columns of A form a basis of $C(A)$
 $\dim C(A) = r$