The Null Space

Note: the non-zero rows of U and R are pivots columns of U and R and hence are linearly independent. This gives us two possible bases:

$$\boxed{\begin{array}{c}\text{The non-zero rows of } U \text{ or of } A \text{ form a basis of } C(A^T) \\ \dim C(A^T) = r = \text{ rank}(A)\end{array}}$$

Consider next the **null space** $N(A)$. For the A above we have found the solutions of $Ax = 0$ to be:

$$x = v \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\uparrow \qquad\qquad \uparrow$$

"special solutions" of $Ax{=}0$

A **special solution** of $Ax = 0$ is one for which one free variable has value 1 and all others are zero. There are as many special solutions as free variables, $n - r$ in all.