The Row Space

Consider first the **row space** $C(A^T)$:

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix} \Longrightarrow C(A^T) = \text{Span}\{ \begin{bmatrix} 1 \\ 3 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 9 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 3 \\ 4 \end{bmatrix} \}$$

But are all three vectors needed in the span? Since G-E and G-J take rows of A and form linear combos of them, $C(A^T)$ will also be the span of the rows of U or the rows of R:

$$A \xrightarrow{\text{G-E}} U = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{G-J}} R = \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C(A^T) = \text{Span}\{ \begin{bmatrix} 1 \\ 3 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 3 \end{bmatrix} \} = \text{Span}\{ \begin{bmatrix} 1 \\ 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \}$$