A Basis of a Vector Space

Definition: A basis of a vector spave V is a set of vectors that

1. spans V (the set has enough vectors)
2. is linearly independent (not too many vectors)

Observation:

- any span of a collection of vectors is a subspace

  as an example, let $S = \text{Span}\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \end{bmatrix}\}$. Then a general linear combo of vectors in S is again in S:

$$c_1 \underbrace{\left(a \begin{bmatrix} 1 \\ 2 \end{bmatrix} + b \begin{bmatrix} 3 \\ 3 \end{bmatrix} \right)}_{\text{linear combo from } S} + c_2 \underbrace{\left(A \begin{bmatrix} 1 \\ 2 \end{bmatrix} + B \begin{bmatrix} 3 \\ 3 \end{bmatrix} \right)}_{\text{linear combo from } S}$$

$$= \underbrace{(c_1 a + c_2 A) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + (c_1 b + c_2 B) \begin{bmatrix} 3 \\ 3 \end{bmatrix}}_{\text{another linear combo from } S}$$