Another Example

Ex: Show that $\mathbb{R}^2 = \text{Span}\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\}$

The question here is: given x_1 and x_2 can we find c_1 and c_2 such that

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} \ ?$$

$$\Leftrightarrow \left[\begin{array}{cc|c} 1 & 1 & x_1 \\ 1 & -1 & x_2 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 1 & x_1 \\ 0 & -2 & x_2 - x_1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & \frac{x_1+x_2}{2} \\ 0 & 1 & \frac{x_1-x_2}{2} \end{array}\right]$$

We conclude that

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{x_1 + x_2}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{x_1 - x_2}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(check that this is indeed true) and we are done.