Example (continued)

$$\left[\begin{array}{cccc|c} 1 & 3 & 3 & 2 & 0 \\ 2 & 6 & 9 & 7 & 0 \\ -1 & -3 & 3 & 4 & 0 \end{array}\right] \xrightarrow{\text{G-J}} \left[\begin{array}{cccc|c} 1 & 3 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

This shows that $c_1 = -3c_2 + c_4$, $c_3 = -c_4$, c_2 and c_4 arbitrary. Therefore

$$0 = (-3c_2 + c_4)\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_2\begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix} - c_4\begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix} + c_4\begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix}$$

$$= c_2\left(\begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}\right) + c_4\left(\begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} - \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix}\right)$$

The two arbitrary c_i's here indicate that there are two independent linear dependencies as we saw at the beginning of this Part. One arises here by setting $c_2 = 1$ and $c_4 = 0$. The other arises when $c_2 = 0$ and $c_4 = 1$. All others are linear combos of these two as shown above.