Testing for Linear Dependence Among Column Vectors

In $\mathbb{R}^n$ every linear combo problem is equivalent to a linear system. If $v_1, v_2, ..., v_k$ are vectors in $\mathbb{R}^n$, then

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0 \iff \begin{bmatrix} | & & | \\ v_1 & \cdots & v_k \\ | & & | \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_k \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\iff \left[\begin{array}{ccc|c} v_1 & \cdots & v_k & 0 \end{array}\right]$$

$$\iff \text{use G-E or G-J to solve for } c_1, ..., c_k$$

Note:

- k pivots in the matrix $V = \left[\begin{array}{ccc|c} v_1 & \cdots & v_k & 0 \end{array}\right]$ implies $c_1 = 0, ..., c_k = 0$
- less than k pivots means there are free variable and so non-trivial solutions for the c_i's